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# THE HILGER MICROSCOPE INTERFEROMETER*

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*ABSTRACT.* The paper relates to an interferometer for measuring the aberrations of microscope objectives—a development of the Hilger interferometers for testing camera and other lenses. A number of possible arrangements are discussed and an actual instrument described. The routine of setting up an objective for test is also detailed.

THE interferometers designed by the writer for the testing and correction of prisms, of lenses for axial pencils, and of camera lenses for both axial and oblique pencils, have been described (see bibliography at end of paper), and it is desirable for those who wish to take up the use of the Hilger microscope interferometer to make themselves acquainted with Refs. 3 and 6 (or 8, which is a reprint of 6).

## PRINCIPLES OF CONSTRUCTION OF THE MICROSCOPE INTERFEROMETER

The microscope interferometer is a development of the lens interferometer (see Ref. 3), which is shown in diagram in Fig. 1. In the latter instrument a collimated beam of monochromatic light is separated into two beams at the transmissively silvered surface of a plate of plane parallel glass $K$. The transmitted beam, which may be called briefly the test beam, passes through the lens $T$ under test, and is reflected back from the surface of the convex mirror $U$, which is so disposed as nearly to coincide with the approximately spherical wave front of the light as it converges after passage through the lens $T$. Thus after reflection and passage back through the lens under test the wave front of the returning beam is approximately plane.

The second beam, which may be called the comparison beam, is reflected back along its own path by the mirror $G$, so that the two beams recombine at the silvered surface of the plate $K$, and pass on together through the lens $E$ which concentrates

* British Patent No. 103832, 1916, and corresponding patents in other countries. Also additional patent applied for.

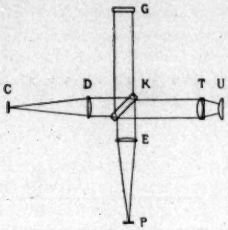

Fig. 1

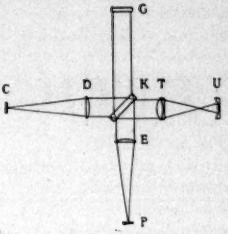

Fig. 2

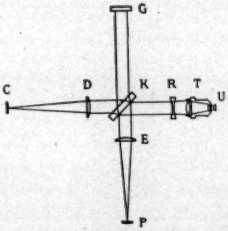

Fig. 3

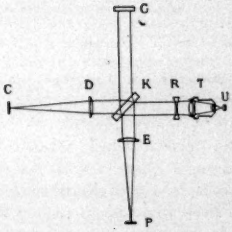

Fig. 4

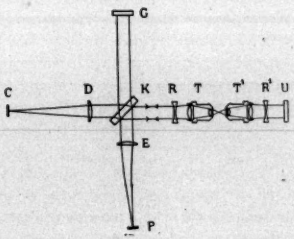

Fig. 5

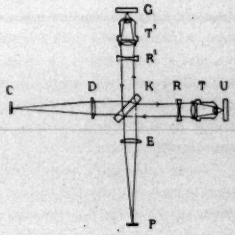

Fig. 6

them on the eye of the observer situated at *P*. Under proper conditions (for which Ref. 3 may be consulted) the observer then sees an interference pattern apparently located on the surface of the lens under test, and this pattern is a contour map, to a scale of half wave lengths of the light used, of the aberrations of wave surface occasioned by passage of a plane wave through the lens *T*. The arrangement may be modified by the use of a concave mirror instead of the convex one, as shown in Fig. 2.

If such an instrument be provided with sufficiently delicate adjustments for focussing the objective under test (namely, moving it to and from the mirror *U*), and for moving the mirror *U* laterally, it becomes immediately suitable for exhibiting the aberrations possessed by a microscope objective when the latter is focussed to produce its real image at infinity. The objective is represented by *T* in Figs. 1 and 2. If, as is customary, the objective is intended to produce a real image at a finite distance, such a condition is simulated by the introduction of a negative lens *R* (see Figs. 3 and 4), corrected for spherical aberration and called, conformably to the nomenclature of the microscopist, the tube length lens. This lens must give to a parallel beam a divergence exactly corresponding to the convergence of the beam which obtains in the intended use of the objective.

The polishing of a concave mirror (Figs. 2 and 4) with sufficient precision not to introduce objectionable aberrations of its own becomes very difficult in the case of objectives of high numerical aperture, while if a convex mirror be attempted (Figs. 1 and 3) it needs to be of very small size, and is on that account very difficult to produce accurately by ordinary procedure. A small drop of mercury may be used as a convex mirror*. In this case also the observer has no direct way of assuring himself that the surface of the drop is sufficiently spherical, and other modifications have therefore been devised, whereby the use of spherical mirrors can be obviated.

Figs. 5 and 6 show arrangements which may be used where the observer is satisfied if he can compare the objective to be tested with a second one (not necessarily identical) which he regards as a standard. In Fig. 5 the standard objective *T'* is in line with that under test, together with a second tube length lens *R'*, while in Fig. 6 the standard objective and its tube length lens are put in the comparison beam. In the arrangement of Fig. 5 the interference pattern represents the sum of the aberrations of the two objectives, while in Fig. 6 it represents their difference. In either case flat mirrors suffice in both the test and comparison beams. It will be seen that in Fig. 6 each ray returns, not along its first course, but along another course axially symmetrical with the first. This has the disadvantage that only in the case of an objective whose aberrations are axially symmetrical does the interference pattern represent the aberration truly, the

Fig. 7

* See Note (*a*) for method of producing.

aberration of wave front shown at two axially symmetrical points (*a* and *a'*, Fig. 7 *A*) being in each case that due to the passage of a ray through *a* in one direction and *a'* in the other direction. Thus a distribution of aberration, due, for instance, to a local defect truly as shown in Fig. 7 *B*, would appear in the interference picture as in Fig. 7 *C*. The arrangement in Fig. 6 has the advantage that, the optical elements traversed by the beams in the test and the comparison beams being more nearly alike than in any of the other arrangements shown, it is possible, for reasons that will be referred to later, to get more illumination. This is true in an enhanced degree in the arrangement of Fig. 8, where the two paths have been made equal by the compensating plate *K'* (identical with *K*). Where, then, it is desired to get photographic records of aberrations of a number of objectives of similar focal length, the arrangement of Fig. 8, supplemented by an ocular examination for non-symmetrical faults by the arrangements of Figs. 3, 4 or 5, has much to recommend it.

Instead of the comparison objective *T'*, tube length lens *R'*, and mirror *G* in Fig. 6, a lens *W* of comparatively long focus (50 or 100 mm. for instance) corrected for spherical aberration, and with a flat mirror in its principal focus, can be used, as in Fig. 9.

Fig. 10 shows an arrangement in which no curved surfaces come into question other than those in the tube length lens and those of the objective under test. Here the lens and mirror *W* and *G* in Fig. 9 are replaced by the prism *X* in Fig. 10. *X* is a tetragonal or "corner cube" prism (or its equivalent), shown separately in Fig. 11. Such a prism has the well-known property that every ray incident upon it is sent back parallel to its own path*. It has also the further property that the incident and reflected rays are precisely symmetrical with each other relatively to that ray which meets the apex of the prism. This property does not seem so well known, and a brief demonstration of it is given in Note (*b*). It will be seen then why such a tetragonal prism can be used, as in Fig. 10, in place of the object glass with mirror in its focus, shown in Fig. 9. The tetragonal prism has the disadvantages that it is difficult to make with sufficient accuracy, and that the edges of the right angles and the images of those edges divide the field into six parts by lines which, though fine, are by no means invisible, and thus slightly hinder the observation of the interference effects. On the other hand, the tetragonal prism has the advantage that it is somewhat easier to get into correct adjustment than are the corresponding lens arrangements.

Another arrangement which is of interest, although presenting no practical advantage, consists in the use of a very small tetragonal prism in place of the convex or concave reflecting mirror in Figs. 3 and 4.

### *Arrangements obviating the necessity of the Tube Length Lens*

It is possible to avoid the use of the negative tube length lens by placing the eye at the position where the objective forms in practice its real image. In this

* L. Silberstein, *Phil. Mag.* 32 (1916) 487; Beck, *Z. f. Instrk.* 7 (1887) 385; Grubb, British Patent No. 21856/1903; Zeiss, British Patents Nos. 5499, 5500/1906.

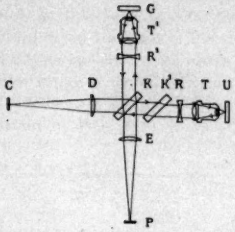

Fig. 8

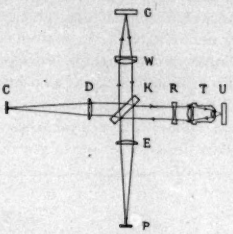

Fig. 9

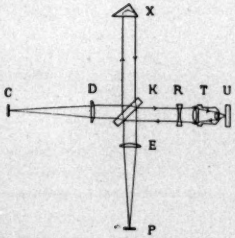

Fig. 10

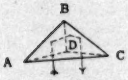

Fig. 11

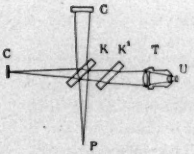

Fig. 12

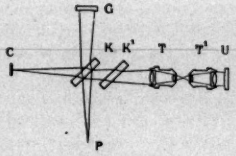

Fig. 13

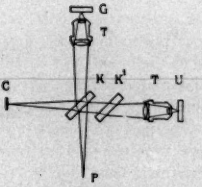

Fig. 14

case, in order that the rays in the comparison beam may have a like convergence, a concave mirror is used where a plane one would otherwise be used, or some other device to cause the desired convergence. The lenses $D$ and $E$ are not required.

Some of the possible arrangements including this modification are shown in Figs. 12, 13 and 14, corresponding with those in Figs. 3, 5 and 8. It is necessary in all these three cases to introduce the compensating plate $K'$.

Of all the above arrangements, the three which have been selected for the standard model instrument are those shown in Figs. 3, 9 and 10.

## Description of the Apparatus

### *Light Source*

The light used must consist of a limited number of very homogeneous radiations. The following are suitable radiations for the purpose:

| | Wave length | Source |
|---|---|---|
| Mercury green ... | 5461 | Cooper-Hewitt mercury vapour lamp. |
| Mercury yellow ... | {5770 / 5791} | " " " " |
| Mercury violet ... | 4359 | " " " " |
| Hydrogen red ($C$) ... | 6563 | Hydrogen vacuum tube. |
| Hydrogen green ($F$) | 4861 | " " " |
| Hydrogen violet ($G'$) | 4341 | " " " |
| Neon yellow... ... | 5853 | Special neon lamp*. |

The use of several of these radiations permits the measurement of chromatic aberration to a very high degree of accuracy.

### *Apparatus*

A view of the apparatus is shown in Fig. 15. The light from the source is reflected by an adjustable mirror $A$ through the condensing lens $B$, by means of which it is condensed on the aperture of the diaphragm $C$. The diverging beam of light is collimated by an achromatic lens $D$, and falls as a parallel beam on a plane parallel plate $K$, the second surface of which is silvered lightly so that a part of the light is transmitted and part reflected†. The major part should be reflected.

The mounting of the plane parallel plate $K$ is shown separately in Fig. 16. It can be set in two alternative positions on the interferometer. The normal position is shown in Fig. 15. The second position, required only in adjusting the apparatus, is at 90° to the first, and requires the leg $L$ (Fig. 16) to be pivoted round in order that it may find a bearing on the interferometer. It will be noted that two milled head screws $H$ and $H'$ are provided for adjusting the inclination of the plate about horizontal and vertical axes. The pivoting axes are provided at the mirror end of the mount by two pairs of steel balls and an intermediate ring, all maintained in their positions by the single spring $J$, which also keeps the arm $A$ against the points of the adjusting screws $H$ and $H'$.

One part of the light passes through the negative tube length lens and microscope objective and on to whatever reflecting device is in use, mounted on the mechanical stage. The microscope objective and the tube length lens are mounted

* Made for Adam Hilger, Ltd., by the General Electric Company, England.

† See Note (*c*) for a suitable method of partial silvering.

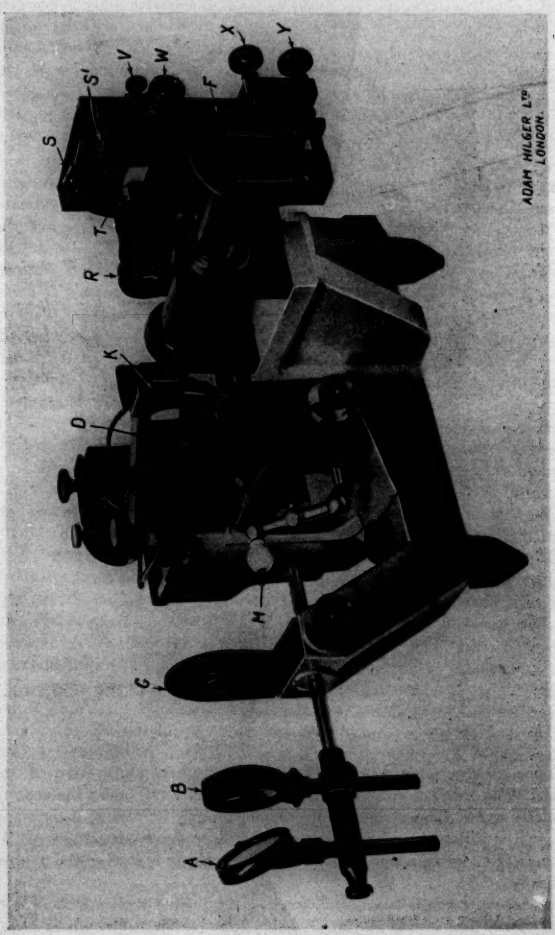


Fig. 15

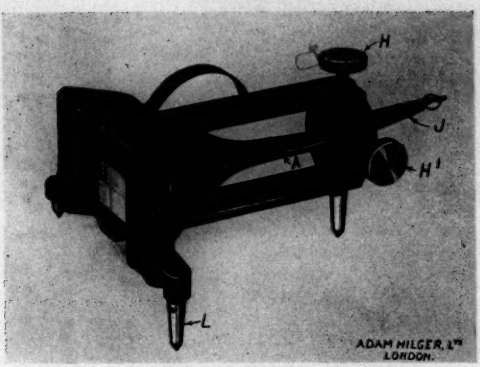


Fig 16

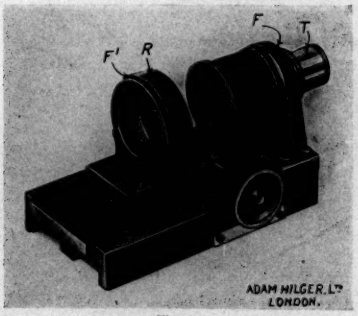


Fig. 17

on the same carriage (shown in Fig. 17) and are very carefully fixed so as to be co-axial. The carriage moves along accurate steel ways under the action of a micrometer screw. This screw can be rotated directly by means of the handle *M* (Fig. 15) or through the intermediary of a slow movement drum *N*. The greatest care is taken to ensure regularity and certainty in moving the objective longitudinally.

The micrometer screw has 1 mm. pitch. The divided head attached to the screw has 100 divisions, and one turn of the slow motion screw, whose head is also divided into 100 parts, corresponds to one division on the main drumhead.

Thus one division on the drumhead of the slow motion screw corresponds to ·1 $\mu$ (one ten thousandth of a millimetre).

The slides are massive, and of steel*, and the sliding surfaces are worked optically flat.

The slow motion mechanism has a cover, with a window for reading the main drumhead.

The micrometer screw is cut between dead centres and ground at constant temperature. Its position is determined longitudinally by a plate of polished corundum at the end near the handle, which end takes the thrust when the objective is moving towards the mechanical stage. A system of opposing screws enables this plate to be set up perpendicular to the axis of rotation of the micrometer screw, whose end is polished to an approximately flat surface. Thus by reflecting light in the direction of the axis of the screw through the thickness of the corundum block (which is polished on both surfaces to enable the end of the screw to be thus observed), one can observe the interference fringes produced between the light reflected from the thrust-taking surface of the corundum and the polished end of the micrometer screw. Observation in this manner enables the plate of corundum to be made perpendicular to the axis of rotation of the screw with great and ascertainable precision. The other end of the micrometer screw thrusts against the polished end of a fixed screw, by means of which one is enabled to reduce the longitudinal play of the micrometer screw to a minimum, without introducing any longitudinal compression. The carriage is pulled towards the thrust by two spiral springs. But for these two springs the carriage can be removed, by lifting it off its slide, and as easily replaced.

In deciding on the fineness of focussing adjustment which should be aimed at, it was borne in mind that the apparatus might be used, by the removal of some of its parts, as a high accuracy micrograph for the use of ultra-violet light, and provision was made for an accuracy suitable, should a microscope objective of 5 mm. focus and numerical aperture 1·0 at tube length 160 mm. operate with light of wave length 1850 A.U. Such an objective should resolve a separation of ·000093 mm., corresponding to a focussing adjustment of the objective of ·00013 mm.

The tube length lens (see Fig. 17) is adjustable relative to the microscope objective by means of a rack and pinion, and a division in millimetres indicates the distance from the shoulder against which the objective screws to the position which would be occupied by the real image of a point formed by the objective when the latter is so focussed that rays to the left of $R$ are parallel.

### *Mechanical Stage*

The mechanical stage has the usual adjustments in two directions at right angles, but exceptional care has been taken to ensure that the movement should

* The slides were made from forgings of specially selected steel, the preparation and treatment of which to avoid subsequent deformations were carried out for us by Sir W. G. Armstrong, Whitworth & Co., whose long experience in the construction of standard screw gauges enabled them to supply a material excellently well adapted for the purpose in view.

be strictly in one plane perpendicular to the run of the ways. The accuracy of adjustment aimed at is much greater than is usually considered sufficient in the microscope. The movement is produced by screws of $\frac{1}{4}$ mm. pitch which thrust against hardened and polished steel plates; one of these screws can be seen at *W*, Fig. 15. No dovetailed slides are used, the two sliding plates merely bearing in each case against flat surfaces, against which they are maintained by springs (*S*, *S'*, Fig. 15).

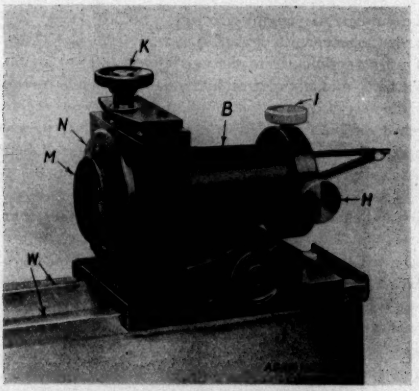


Fig. 18

If required, a very fine lateral adjustment can be obtained by levers (one is shown at *F*, Fig. 15) which clamp on the screws, the said levers being in turn actuated by the fine motion screw heads *X* and *Y*. It is not necessary to have these fine adjustments in the use of the instrument as an interferometer. Where the instrument is intended for conversion into a micrograph a suitable form of condenser can be mounted below the mechanical stage.

The adjustment of the mechanical stage is not intended to have a range of more than about $\pm$ 1$\frac{1}{2}$ mm. from the middle position.

*Comparison Beam Carriage*

This is so arranged that it will take the three forms of reflecting devices, shown in Figs. 3, 9 and 10, these being mounted in tubes which push into a cylindrical fitting in the adjustable body, *B* (Fig. 18), of the carriage. In removing a reflecting device, one has to draw it out by means of the milled ring *M*, while holding the ring *N* in position with the thumb and first finger of the other hand. The middle finger of the latter hand should rest on *B* to steady the carriage.

Two kinds of adjustment are provided, viz. tilting adjustments in azimuth and altitude by the screws *H* and *I* and lateral movements horizontally and vertically by means of the two screws *J* and *K*. The tilting movement is required for the arrangement shown in Fig. 3, both the tilting and lateral movements for the arrangement in Fig. 9, and the lateral movement alone for the tetragonal prism shown in Fig. 10.

The cylindrical sleeve into which these reflecting devices fit pivots upon two pairs of steel balls placed in the neighbourhood of the ring *N* and since the screws *H* and *I* are 9 cm. from the pivoting point and the screws are of $\frac{1}{2}$ mm. pitch, the tilting movement is amply slow enough for convenient adjustment.

The comparison beam carriage fits on ways *W* similar to those of the objective carriage, but no slow movement is required for its longitudinal adjustment. This can be accomplished by hand quite conveniently, and with the necessary precision, adjustment within a millimetre being amply sufficient to attain a position of good visibility of the interference bands.

## Makers' Adjustments

There are certain adjustments for which the makers must take responsibility, but with which the observer should be acquainted. There are two reasons why this is desirable; first, that he may realise the need for them and be prepared to verify them if the instrument should sustain obvious damage or give dubious results; second, that he may know that they have already received the makers' attention and should not be interfered with on a mere conjecture.

### *The Mechanical Stage*

This has been carefully squared-on relatively to the ways, so that not only is the face, against which the microscope slide lies, accurately perpendicular to the ways to about 3 minutes accuracy, but so also is its movement under the action of the mechanical stage adjusting screws.

The movement of the mechanical stage has been tested by mounting on the movable plate one of a pair of Fabry-Perot interferometer mirrors, the corresponding plate of the pair being mounted on the fixed portion of the stage. In this way it has been ascertained that the stage movement is uniform, in one plane, and sufficiently free from "lost time"; also that clamping the slow motion levers, to bring the fine adjustments into play, causes no disturbance of the slide.

The fineness of adjustment aimed at is such that if thus tested the bands of Hg 5461 can be set to any desired position to one-tenth of a band (·027 $\mu$). Such a fineness of adjustment exceeds all present requirements, but may be requisite if the interference instrument should eventually prove to be usable as a microscope interferometer with radiation in the ultra-violet.

### *The Objective and Tube Length Lens Mounts and Carriage*

The face *F* (Fig. 17) of the objective mount and the face *F'* of the tube length lens mount are both made parallel to the face of the mechanical stage to about

three minutes accuracy. The two mounts are made co-axial by screwing in place of the tube length lens cell a plug with a cylindrical fitting, and so adjusting the position of the two mounts that the end of the plug, which projects through the threaded objective mount, is central to it, to within about ·05 mm. The geometrical and optical axes of the tube length lens are made coincident to within about ·025 mm. Thus, if the objective itself be truly mounted, one is assured that objective and tube length lens are co-axial to within the limits indicated by the above figures.

*The Collimator*

The collimator (consisting of the diaphragm $C$ and the achromatic lens $D$, Fig. 15) requires to have its axis perpendicular to the face against which the microscope objective screws. Since the said face has been made perpendicular to the run of the ways (see above) it suffices to set the axis of the collimator parallel to the run of the ways. This is done by setting up an accurate right angle prism resting on the top surface of the ways and perpendicular to the run of the latter. If then light is allowed to fall on the diaphragm, a spot of light, which is an image of the diaphragm aperture, is formed on the diaphragm itself, the image being formed by the light reflected back from the surface of the right angle prism. The position either of the achromatic lens or of the diaphragm is then so adjusted that the back reflected spot falls on the aperture of the diaphragm itself. Simultaneously, the collimator is set for focus, this being done by adjusting the relative positions of diaphragm and achromatic lens so that the back reflected spot is in focus simultaneously with the diaphragm aperture. Observation is made by looking in at the eye end of the interferometer (using an eyepiece) and with the diagonal mirror in the alternative position referred to on p. 194.

## How to Set up and Adjust the Microscope Objective

*Arrangement of Fig.* 3

The method varies somewhat according to the particular arrangement which is in use. In Figs. 3 or 4 the plane mirror $G$ is placed in the comparison beam carriage and the latter brought to such a position that the distance from $G$ to a selected point on the silvered surface of the diagonal mirror $K$ is approximately equal to the distance from the same point to the mirror $U$. It is only when the path lengths in the test and comparison beams are approximately equal that the interference bands are seen distinctly*. The aperture in the diaphragm $C$ (Fig. 15) should be illuminated with suitable light. Placing the eyepiece in the fitting provided at $P$ the spot of light reflected from the mirror $G$ back on to the diaphragm at $C$ will be clearly seen simultaneously with the hole in the diaphragm itself. By tilting the mirror $G$ by means of the screws $H$ and $I$ (Fig. 18), the spot of light should be made to fall on the hole.

Let us suppose that a mercury drop has been selected to act as a mirror, and that such a drop, on a microscope slide, has been placed in position on the mechanical

* For visibility of the bands see Ref. 6, p. 188, and Ref. 9.

stage. If the eye be placed so as to look into the objective from the direction customarily occupied by the condenser, a small bright spot of light will be seen which is the image of the aperture in the diaphragm. Adjust the slide on the mechanical stage so that the drop of mercury obscures symmetrically this spot of light; or alternatively, having adjusted the drop approximately central in position, perfect the centralizing by observation of the drop in the telescope. The drop having been got by these means as nearly central with the objective as possible, cut out the light from the comparison beam by placing a piece of card in front of the mirror *G*. Then it will be found, on slowly focussing the objective by means of the handle (Fig. 17) and observing with an eyepiece at *P*, that a small image of the diaphragm aperture will be seen by reflection at the surface of the drop. To be precise, as one focusses towards the mechanical stage, four small circular spots come into focus in succession. They are:

(*a*) The image of the diaphragm aperture formed upon the surface of the mercury drop.

(*b*) The image of the aperture of the achromatic lens *D* formed upon the surface of the mercury drop.

(*c*) The image of the diaphragm aperture formed at the centre of the mercury drop.

(*d*) The image of the aperture of the achromatic lens *D* formed at the centre of the mercury drop.

It is image (*c*) which is the correct one. This should be got in focus by the handle *M* and central in the field by the mechanical stage screws. On removing the eyepiece and looking in at *P* with the naked eye (the piece of card referred to above being taken away), one will in general see a round field of green light, which represents the surface of the mirror *G*, and near the centre of this circular green field a bright green spot. The objective should now be re-focussed, when the small bright spot will enlarge in size until it reaches a maximum diameter which corresponds with the useful aperture of the back lens of the microscope. At this position the interference pattern will be seen.

### *Arrangement of Fig.* 9

It is necessary in the arrangement shown in Fig. 9 that the beam passing through the achromatic lens reflector should be parallel to the axis of the lens. This lens and mirror arrangement will act, even when it is tilted, as a reflector which sends incident rays back in a direction parallel to themselves. It is necessary, therefore, to have some independent assurance that the incident rays are parallel to the axis of the lens. This can be made sufficiently certain by first putting in its fitting mirror *G* (Fig. 3), and adjusting it so that the spot of light is reflected back by it on to the diaphragm, as described in the adjustment for the arrangement of Fig. 3. If the achromatic lens and mirror reflector are then carefully put in place of the mirror, the condition in question will be fulfilled with sufficient exactness. The tilting adjustment, of course, should not be used after the interchange is made.

On looking in at *P* one will observe an illuminated field due to the light from

the comparison beam. If the objective is approximately in focus on the surface of the plane mirror, a small bright spot is seen, which can be enlarged to its maximum diameter by focussing the objective. When this position has been found, the reflector in the comparison beam must be adjusted laterally by means of the screws *J* and *K*, Fig. 18.

When the position has been found in which the returning rays in both the test and comparison beams return to the same point of the diagonal mirror, the interference pattern is seen, but unless one has some indication that this correct adjustment is being approached, the search for the correct position may be indefinitely prolonged.

For this reason two steel points, mounted on brass plates, are provided—the one to stand on the ways of the comparison beam carriage, the other to stand on the microscope objective carriage. Each of these points should be shifted sideways until its shadow is in line with the pointer itself and the eye placed at *P*. When this is done for both pointers, they will in general not appear to occupy the same position. The one in the comparison beam should then be shifted sideways until it lies in the same line as seen from the eyepiece and *P*, no eyepiece being used, as the image of the pointer in the test beam. If then the reflector mount is shifted horizontally and vertically by means of the screws *J* and *K* until the tips of the two pointers and those of their images all lie in the same straight line, the interference appearances will be seen. It is assumed of course that the reflector carriage has been first put in such a position that the length of path of the rays in the test and comparison beams is approximately the same.

*Arrangement of Fig.* 10

The adjustment of the tetragonal prism reflector is very similar to that of the achromatic lens and mirror reflector. The carriage should be placed in such a position along the slide that the apex of the tetragonal prism is about the same distance from any selected point on the diagonal plane as is the plane glass reflector in the test beam. Strictly speaking, one should, in measuring these distances, take account of the thickness of glass traversed, but it is seldom necessary to do so since interference bands will be seen with a rough adjustment of distance, and once the bands are seen, the shifting of the comparison beam carriage to the position where the bands are most distinct is a very simple matter. The tetragonal prism requires no tilting adjustment; all that is necessary is that it should be adjusted laterally with the two screws. Its correct position is such that the six dividing lines by which its field is divided into six segments are symmetrically disposed relative to the effective aperture of the back lens of the microscope objective; thus the adjustment of the tetragonal prism is rather simpler than that of either of the other devices.

*The Interference Appearances and their Interpretation*

The interferometer here described is designed for examination of the aberrations of axial beams. Designs have been got out for testing oblique pencils, but the apparatus then necessarily becomes more complex. The remarks given in a

previous paper (see Ref. 6, p. 182) with regard to appearances seen on the camera lens interferometer would be broadly applicable to a microscope interferometer such as here described but possessing modifications enabling the wave surface aberrations for oblique pencils also to be examined.

## NOTES

(a) *Preparation of Mercury Drops.*

It is necessary that the drop should be small and free from grease and dust. Such drops may be prepared in the following way:

Place a small amount (say $\frac{1}{2}$ c.c.) of clean mercury in a test-tube. Pour on it a few c.cs. of strong sulphuric acid in which as much chromic acid has been dissolved as it will take up. Shake for about 3 seconds, and then put under a water tap to wash the acid away. Then rinse with distilled water and pour off as much of the water as possible. Now with a pipette with rubber bulb (the nozzle of which has itself been cleaned with the sulphuric and chromic acid mixture, washed and drained dry) transfer a drop of the mercury to a similarly cleaned microscope slide. Allow a second clean slide to fall on the first, when the mercury drop will be broken into a number of small ones suitable for use in the interferometer.

Another way (described by Barnard) is to hold a microscope slide over the mouth of a test-tube in which mercury is boiling until drops of a suitable size (say $\frac{1}{4}$ mm. diameter or less) are formed by condensation.

(b) *Tetragonal Prisms.*

Two parallel rays in a homogeneous medium are the same distance apart after any number of reflections of the two rays at plane surfaces. For let *AOB*, *A'O'B'*, Fig. 19, represent two such rays, each once reflected at a plane surface *EDFG*. If we imagine the whole system of rays rotated through 180° about an axis perpendicular to *EDFG* through a point *P* midway between *O* and *O'*, it is obvious that the so rotated system of rays will occupy the positions occupied by the rays before such rotation. Thus the distance apart of parallel rays after one plane reflection is the same as before reflection, and if the same after one reflection, then the same after any number.

It readily follows that the incident ray which within the tetragonal prism has the direction *KL* (Fig. 20) will be symmetrical with the reflected ray *PS* relatively to the ideal apical ray *MN* (which we can also assume to have suffered three reflections and returned along its own path). Obviously the same is true concerning the corresponding rays *FK*, *ST*, *QM*, external to the prism, no matter how the latter be tilted.

(c) *The Deposition of a Transparent Film of Silver on Glass.*

*Solutions*

Prepare the following stock solutions:

*A*

10 per cent. silver nitrate.

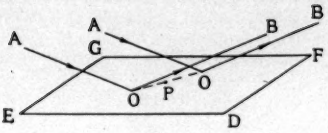


Fig. 19

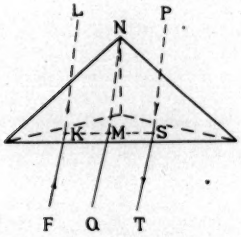


Fig. 20

*B*

40 per cent. formaline.

*C*

| | | | |
|---|---|---|---|
| Granulated sugar | ... | ... | 400 grms. |
| Alcohol | ... | ... | 200 c.c. |
| Nitric acid | ... | ... | 10 c.c. |

Made up with distilled water 2000 c.c. (this should be allowed to stand two weeks before using).

*D*

| | | | |
|---|---|---|---|
| Chromic acid | ... | ... | 250 grms. |
| Sulphuric acid | ... | ... | 1500 c.c. |

*Procedure*

The glass plate is placed in a glass dish, and cleaned first by strong nitric acid swabbed over the surface by small wads of cotton wool twisted round the end of a stock or glass rod, and then by allowing the glass to stand in some of *D* solution for 5 minutes. Pour off solution, rinse the plate thoroughly by a stream of running water, lifting the plates with a glass rod to allow acid to escape from beneath the glass plate. Take 20 c.c. of *A* solution, add ammonia until the precipitate is *just*

redissolved, add silver nitrate solution (any strength) until the liquid is a faint straw colour. Make up to 100 c.c. with distilled water.

*Reducing Solution*

5 c.c. of *B*
5 c.c. of *C* } mixed.

The mirror can be suspended with the face either upwards or downwards in the ammoniacal solution of silver. Then add the reducing solution. Keep the solution in constant motion until it becomes reddish in colour. Pour off the solution and put in a second quantity of the ammoniacal solution of silver *without* any reducing solution. Allow the mirror to remain until the required density of deposit is obtained—this will take a few minutes only. The plates can then be rinsed with distilled water and dried. No polishing is required, the deposit being bright and uniform.

For the interferometer it is better to have the deposit of silver rather dense.

For the fully silvered mirrors any of the usual silvering processes may be used.

(d) *Microscope Tube Lengths.*

There are two tube lengths, 160 mm. (Continental) and 250 mm. (English). The words "tube length," or more precisely "mechanical tube length," mean the distance from the shoulder of the objective to the tube end against which the eyepiece rim rests. There is therefore no precise connection between back focus and tube length.

In the microscope interferometer the negative tube length lens is mounted so that the distance from the objective shoulder to the back focus can be altered from about 145 mm. to 175 mm.; or if a 250 mm. tube length lens be supplied, an adjustment from about 235 mm. to 265 mm. is provided.

## BIBLIOGRAPHY OF HILGER INTERFEROMETERS

1. Patent No. 103832/1916 and corresponding foreign patents, covering the prism and lens interferometers of which one form is described in Ref. 3.
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3. "Interferometers for the Experimental Study of Optical Systems from the point of view of the Wave Theory." By F. Twyman, *Phil. Mag.* 35 (1918) 49.
4. "On the Use of the Interferometer for Testing Optical Systems." By F. Twyman, The Twenty-First Annual Traill-Taylor Memorial Lecture, delivered before the Royal Photographic Society of Great Britain, 15th October, 1918.
5. "Correction of Optical Surfaces." By F. Twyman, *Astrophys. Journ.* 48 (1918) 256.
6. "An Interferometer for Testing Camera Lenses." By F. Twyman, *Trans. Opt. Soc.* 22 (1920–21) 174.
7. "The Testing of Microscope Objectives and Microscopes by Interferometry." By F. Twyman, *Trans. Faraday Soc.* 16 (1920) 208.
8. "An Interferometer for Testing Camera Lenses." By F. Twyman, *Phil. Mag.* 42 (1921) 777 (reprint of Ref. 6).
9. "Fringe Systems in Uncompensated Interferometers." By J. Guild, *Proc. Phys. Soc.* 33 (1920–21) 40.

## DISCUSSION

Mr J. Guild: Mr Twyman is to be congratulated on adding one more to his series of valuable adaptations of the Michelson interferometer.

I am surprised to find that a mercury globule proves to be sufficiently truly spherical for the purpose to which it is put.

I am interested to see that Mr Twyman makes use of tetragonal prisms as mirrors in one of the arrangements described. Some years ago, before I had the opportunity of using one of Hilger's interferometers, but after having been shown them in operation by Mr Twyman, I attempted to construct an instrument using tetragonal prisms in order to avoid the need for great rigidity and accurate adjustments. I gave up the attempt, chiefly because of the difficulty that the outward and return rays are not coincident but on opposite sides of the apex, so that the device could only be used, as pointed out in the present paper, to indicate defects of a symmetrical nature.

Mr J. W. Hasselkus: The microscope interferometer serves purely for measuring the aberration in the wave front, and has not yet become a universal workshop and manufacturing instrument. I had hoped that Mr Twyman would mention the work he has done in making the results of an interferometer examination intelligible to computers.

Dr J. S. Anderson: It would be of interest to many Members of the Society if Mr Twyman could supply interferograms of typical microscope objectives of different makes.

Mr T. Smith: I would like to ask how Mr Twyman knows when a microscope objective is exactly focussed for a given tube length. The importance of securing the proper adjustment is illustrated by the following result. I made calculations on a $\frac{1}{8}$ inch objective of aperture $f/1$, which was entirely free from aberrations for the correct object position, and found that if the object were displaced $\frac{1}{10}$ mm. from this position along the axis, the marginal rays, instead of coming to a focus on the axis, were actually diverging from the axis in the image space.

Mr G. W. Gosling: All medium and high power microscope objectives are corrected for certain thicknesses of cover glass. In testing an objective on the interferometer what method does Mr Twyman adopt to compensate for the thickness of cover glass for which the objective is corrected? In practice, of course, any slight deviation from the correct thickness is overcome by altering the tube length.

Dr J. W. French: The instrument that has been described to us to-night, although for a specific purpose, namely the testing of small lens combinations such as microscope objectives, is clearly a result of the many years of work that Mr Twyman has devoted to the development of interferometers. I personally can testify to the excellent results that Mr Twyman has been able to obtain in the

figuring of large optical parts by means of his interferometer methods. At one time, when optical glass was perhaps not quite so good as it is to-day, the figuring of our large elements was done by Mr Twyman's firm. At the present time, for the comparatively small amount of retouching that is necessary, other methods of testing such as the Foucault suffice, and it is remarkable how expert opticians become with practice. No doubt experience plays an even greater part in the use of the interferometer and I would be glad to know if, with practice, the operator can to any extent interpret the appearance in the field of view and allocate the defects. To do so must always be difficult, in view of the large number of variables, such as defects of homogeneity, irregularities of surface, errors in thicknesses, and spacing, and errors of centering, not only of the individual lenses but also of the axis of the system as a whole.

It would be very interesting to have at some future date, after the instrument has been in use for some time, a report as to its performance.

Mr Twyman: Mr Guild expresses surprise to find that mercury globules prove sufficiently spherical for the purpose required. I have used no direct means to measure their shape, and my justification for supposing certain of them to be spherical enough lies in the observation that the appearances seen with them are axially symmetrical, unless the objectives themselves have asymmetrical defects (distinguishable by turning an objective to various orientations about its axis). It is true that there are plenty of the mercury globules which are not spherical, but as a rule they are then very distinctly bad.

Mr Hasselkus has referred to the work recently done in translating the results of interferometric examination into the system of geometrical aberrations used by computers. The work has been carried out under the auspices of the British Scientific Instrument Research Association, and although that Association has kindly given permission to publish the results after they have been circulated to the members, a few weeks will pass before this is done. An account of the work will then be offered to this Society.

I should have been very glad to show interferograms of typical microscope objectives as suggested by Dr Anderson, but unfortunately I have none to show. The apparatus described was only in my hands for a few weeks after its completion, and the time I spent with it was chiefly occupied in its adjustment. It is now in the hands of Mr J. E. Barnard, for whom it was made. If it has useful possibilities, I am sure there is no one better qualified to develop them, and in that case we shall no doubt hear about them in due course.

Mr Smith has asked how I know when a microscope objective is focussed for a given tube length. In practice the question would arise in one of two ways. Either the computer or maker would state the tube length for which the objective is corrected, in which case the tester could try whether with that tube length the objective is in fact good, or else the tester would set himself to find on the interferometer for what tube length the correction was best, altering the position of the tube length lens until getting the best result, making also (if such a refinement should be desired) the correction found by experience to be rendered desirable by

the eyepiece to be used. Indeed, there is no reason why the eyepiece should not be tested on the interferometer too.

Mr Gosling has asked whether allowance is made for the thickness of the cover glass to be used. The reply to his query is much the same as that to Mr Smith's. One can test the objective with a cover glass of any thickness, or with no cover glass at all; or, alternatively, one can allow for the effect of the cover glass in accordance with experience previously obtained with the interferometer. If the maker temerariously states the thickness of cover glass that should be used, the statement can be checked; and if he does not, the best thickness of cover glass can be found by trial.

I agree with Dr French as to the sensitiveness of the old Foucault method as a test for defects of optical systems. Indeed, as has been stated here on a previous occasion, I believe by Mr Smith, for systems including telescopes and prisms there is little to choose in sensitiveness between the Foucault method and the interferometer method, always supposing the former is driven to its extreme capability. The disadvantages of the Foucault method are twofold:

1. It renders visible only the rate of change of the errors relative to one direction.

2. It provides no means of measurement, and does not discriminate between the magnitudes of errors which exceed a particular amount.

I take it that Dr French is not referring to the use of the Foucault method for the test of microscope objectives. I cannot precisely picture any very favourable form in which the principle could be applied to such lenses.

In reply to Dr French's last question, I could not with my present experience say what part of an observed aberration of wave surface might be due to faults of computation and what part to errors of manufacture, except in the case of errors of centering, which of course are distinguished by being asymmetrical.

# ON THE FORM OF THE WAVE-SURFACE OF REFRACTION

By A. WHITWELL, M.A., A.R.C.Sc.I.

*MS. received, 6th March, 1923. Read and discussed, 12th April, 1923.*

*ABSTRACT.* The graphic method used by Prof. R. W. Wood for investigating the wave-surface of reflection is applied to the wave-surface of refraction. A series of wave-surfaces is drawn for each of a number of plane or spherical refracting surfaces, the incident wave being plane or spherical.

It is shown that in each series the wave-surfaces are successively of one of the following types:

1. Saucer.
2. Saucer with inturned edges.
3. Closed surface.
4. Goblet.
5. Basin.

The forms of these types are briefly described and some properties of the wave-surfaces are discussed.

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In his *Physical Optics* Prof. R. W. Wood investigates the form of the wave-surface of reflection by a graphic method and illustrates the various forms of wave-surface by photographs of a sound wave-surface generated by a disturbance in air produced by an electric spark.

It occurred to me, in view of recent papers read before this Society by Mrs Griffiths* and Dr Martin†, that it would be of interest to apply Prof. Wood's graphic method to the investigation of the form of the wave-surface produced when a plane or spherical wave is refracted by various forms of refracting surface or by a lens and I have accordingly drawn on a large scale a series of wave-surfaces for each of a number of refracting surfaces.

Fig. 1 shows the refraction of a plane incident wave $I$ in air by a convex spherical air-glass refracting surface $R$; the index of refraction is taken as $\mu = 1{\cdot}5$ for convenience of drawing. The first thing done was to draw a number of incident rays and the corresponding refracted rays by a geometrical construction devised by me and described in the *Optician* of May 4th, 1917, or by means of a little tool called a "Refractograph" described in the *Optician* of July 28th, 1922. The next step was to draw the trace of the orthogonal or orthotomic surface, shown as a dotted line and marked $O$. This was done by producing the refracted ray backwards and setting off from the point of emergence a distance $bc$ such that $\mu\,(bc) = ab$, where $ab$ is the distance along the incident ray between the plane wave front and the refracting surface; in other words, $bc$ was made $\frac{2}{3}$ of $ab$. The various points $c$ were then joined

* *Trans. Opt. Soc.* 21 (1919–20) 87. † *Trans. Opt. Soc.* 23 (1921–22) 63.

up by a smooth curve $O$. The next step was to take any arbitrary length, say $cd$, in the compasses and set it off along the refracted ray, starting from the orthogonal surface. By joining up all points like $d$ we get one of the series of wave-surfaces in the glass. The optical length of the path $abd$, of which $ab$ is in air and $bd$ in glass, is the same as the optical length of the path $cd$, considered to be wholly in glass; consequently as all the transverse disturbances or displacements associated with the propagation of light are in the same phase in the incident wave-front they will also be in the same phase when they reach the wave-surface. The first wave-surface $S_1$ is so drawn that the path of the marginal ray is wholly in air. The curve $S_1$ is

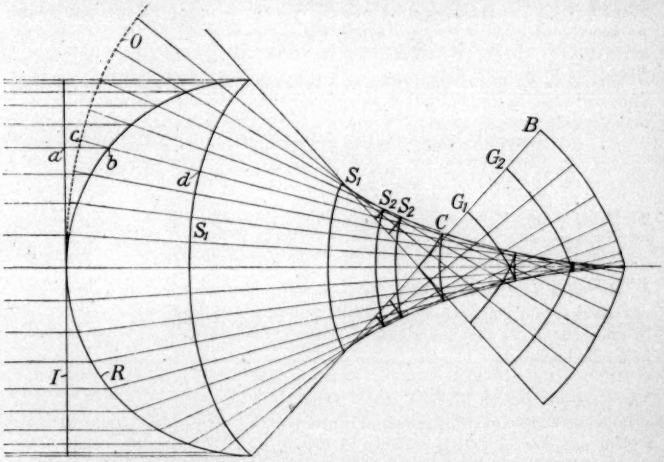



Fig. 1. Wave-surface of refraction. Plane wave, convex surface. $\mu = 1{\cdot}5$.

of course the section of the wave-surface by the plane of the paper and if it be supposed to rotate round the axis it will trace out in space the true wave-surface which I propose to call of the "saucer" type. The wave-surfaces are of the saucer type and are limited in extent by the refracted marginal rays till we get to the point where the caustic begins; beyond this point the *edges* of the saucer $S_2$ begin to turn inwards and are concave towards the incident light, the *middle* portion being convex towards the incident light as before. The cusps all touch the caustic surface. The inturned edges of the saucer correspond with rays which have passed through the primary focus on the caustic and are diverging. The wave-surfaces are of the "saucer with inturned edges" type $S_2$ till we reach the point where the two extreme marginal rays intersect one another on the axis. At this point the wave-surface $C$ becomes a completely closed surface like a cone with a dished bottom. Proceeding

further we get a type of wave-surface which is marked *G* and which may be termed the "goblet" type. In this type the bowl of the goblet is open towards the incident light and the base of the goblet is a closed surface like the surface marked *C*.

As we proceed further to the right the bowl of the goblet increases in size and the base diminishes until when we reach the focus for paraxial rays the base has disappeared altogether and the goblet has become what I may call a "basin" type *B*.

This succession of types of wave-surface, viz.:

1. Saucer,
2. Saucer with inturned edges,
3. Closed surface,
4. Goblet,
5. Basin,

will be found to be common to all refracting systems in which the surfaces are surfaces of revolution of a conic section. I am speaking now only of incident waves which are plane or spherical with the radiant point on the axis.

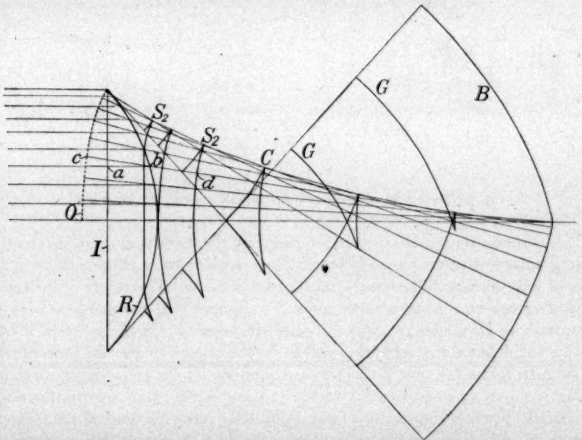



Fig. 2. Wave-surface of refraction. Plane wave, plano-convex lens or concave sphere.

These wave-surfaces are really the envelopes of a large number of circles, of the same radius, the centres of which are all on a previous wave-surface or on an orthotomic surface. The construction described above is merely an easy way of drawing these envelopes.

Fig. 2 shows the wave-surfaces produced by the refraction of a plane wave by a plano-convex lens or by a concave glass-air spherical surface. The surface *I* may be regarded either as an incident plane wave-surface in glass or as the first

surface of a plano-convex lens. It will be noticed that the distance *bc* set backwards from the point of emergence *b* is now $\mu$ or 1·5 times the distance *ab* which the wave travels in glass from the wave-surface *I* to the refracting surface *R*. The first wave-surface drawn is of the "saucer with inturned edges" type. The saucer has a flat bottom, part of the wave-surface being still within the glass. The surfaces *C*, *G*, *B* are all as before.

Fig. 3 shows the wave-surfaces produced by the refraction of a spherical wave *I* by a concavo-convex meniscus lens or by a concave glass-air sphere. The curve *I* may be regarded either as the incident spherical wave-surface in glass or as the first surface of a lens.

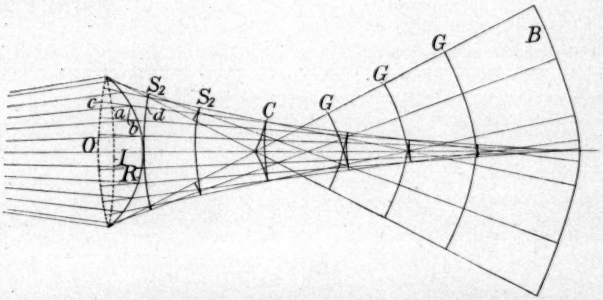



Fig. 3. Wave-surface of refraction. Spherical wave, meniscus lens.

Fig. 4 shows the wave-surfaces produced by the refraction of an incident converging spherical wave *I* by a convexo-plane lens or by a plane surface *R*. The types of wave-surface follow in the same order as before. The centre of the incident converging spherical wave is taken at such a distance that the marginal rays graze the surface as they emerge from the glass; in other words the extreme marginal rays are incident on the refracting surface at the critical angle. In consequence it will be seen that all the wave-surfaces are bounded by the refracting surface itself.

Fig. 5 shows a plane wave *I* refracted at a concave air-glass spherical surface *R*. The parallel beam becomes a diverging beam after refraction and all the true wave-surfaces $S_1$ are of the saucer type. The orthogonal or orthotomic surfaces $OS_1$, $OS_2$, *OC*, *OG*, etc., shown in dotted lines, are now exactly similar to the true wave-surfaces shown in previous drawings in which the orthogonal surfaces were of the saucer type. The orthogonal or orthotomic surfaces may be regarded as virtual wave-surfaces.

I have also drawn the wave-surfaces when the refracting surface is generated by the revolution of a parabola, ellipse, or hyperbola round the axis, but it is not necessary to reproduce them here. The sequence of types of wave-surface is exactly the same as in the case of spherical surfaces.

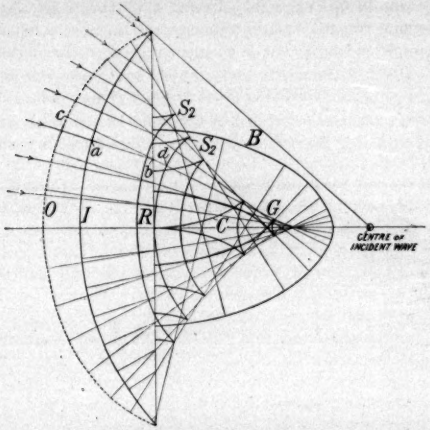



Fig 4. Wave-surface of refraction. Incident wave concave sphere, refracting surface plane, critical angle.

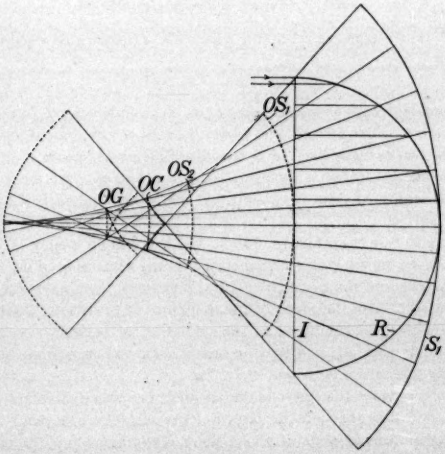



Fig. 5. Wave-surface of refraction. Plane wave, concave surface, glass-air.

In all the examples up to now the refracted wave-surface has a large amount of spherical aberration but no coma or astigmatism. In the two following figures I have taken examples in which coma or astigmatism is present in addition to spherical aberration.

*Spherical Aberration and Coma.*

Fig. 6 shows a plane wave $I$ incident obliquely on the plane face of a plano-convex lens. In drawing the orthogonal surface $O$ we set back from the point of

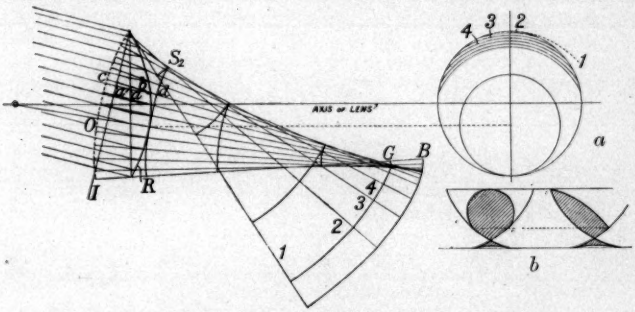



Fig. 6. Wave-surface of refraction. Plain oblique wave, plano-convex lens.

emergence $b$, on the concave refracting surface $R$, a distance $bc$ along the emergent ray such that

$$bc = a'a + \mu\,(ab),$$

so that the optical length from the point $a'$ on the incident wave front to the point $b$ (of which $a'a$ is in air and $ab$ is in glass) is equal to the optical length $cb$, all of which is supposed to be in air. The optical length from the point $a'$ on the incident wave front to the point $d$ on a wave-surface, viz. $a'a + ab + bd$, therefore, is equal to the optical length $cd$ from the point $c$ on the orthogonal surface to the point $d$ on the wave-surface.

As the incident wave-surface is oblique and there is no longer axial symmetry, the wave-surfaces can no longer be generated by the revolution of the curves about the axis of the lens, and the curves $S_2$, $G$, and $B$ must be merely regarded as sections of the wave-surfaces by the plane of the diagram. It is obvious that these curves are identical with those produced by the refraction of a plane wave at a concave spherical refracting surface, as shown in one of the previous figures, but with parts cut off leaving the curves lop-sided.

It is equally obvious that there is one incident ray which, after refraction at the first surface, will pass through the centre of the second surface of the lens. This ray forms a secondary axis about which there *is* symmetry. If the curves $S_2$, $G$, and $B$ be rotated about this secondary axis they will generate the wave-surfaces

but, owing to the fact that the rim of the lens is not symmetrical with respect to the secondary axis, these wave-surfaces of revolution will not be complete but will be truncated except for a small space round the secondary axis where the double folds or bases of the goblet type wave-surfaces are found.

That this is so may be seen by considering the curve $G$ and by supposing it to rotate round the secondary axis with the rays 1, 2, 3, 4, etc., rigidly attached to it. As shown by the end view of the lens (Fig. 6 *a*), the extreme marginal ray 1 in the median plane, or the plane of the diagram, will immediately move off the lens when the rotation begins. This means that there would be no incident ray to correspond with such a refracted ray as 1. On the other hand, the ray 2 will keep on the lens during a certain angle of rotation about the secondary axis in both directions as shown by the curve marked 2 on the end view of the lens. So long as a ray keeps on the lens, so long will there be a real portion of the wave-surface corresponding with it. The rays 4, 5, 6, etc., as they are nearer and nearer to the secondary axis, will continue longer and longer on the lens until at length they keep on the lens for the complete rotation, as indicated by the eccentric circle on the end view. It follows, therefore, that a portion of the wave-surface corresponding with a curve such as $G$ (Fig. 6) is a complete surface of revolution whilst the remainder is a truncated surface of revolution. In Fig. 6 *b* I have sketched two views, taken at right angles, of a goblet type of wave-surface. The shaded parts roughly indicate the part of the wave-surface which actually exists, the plain parts being cut off by the rim of the lens. The true shape of the shaded parts might have been accurately projected by the methods of descriptive geometry.

If a small stop be placed in front of the lens in such a position that its image with respect to the first surface is at the centre of the second surface it is clear that only those rays which are symmetrical with the secondary axis will get through and consequently the lop-sided tail will be cut off and the coma will be corrected. This position of the stop also corrects the astigmatism of the oblique pencils.

### *Spherical Aberration and Astigmatism.*

Fig. 7 shows a plane wave refracted at a glass-air toric surface; it is an orthographic projection, the axis of the toric surface being the $xy$ line or the intersection of the two planes of projection. The part of the drawing above the axis is therefore the elevation of half the vertical sections of the wave-surfaces and the part below the axis the plan of half the horizontal sections of the wave-surfaces. The section of the toric refracting surface by the vertical plane, or by the plane of the diagram, is a circle of radius $R_1 = 20$ cm. The section of the toric surface by the horizontal plane is a circle of radius $R_2 = 15$ cm. The two powers of the toric surface for $\mu = 1{\cdot}5$ are $+ 2\frac{1}{2}\,D$ and $+ 3\frac{1}{3}\,D$, so that on the axis there is $\frac{5}{6}\,D$ of astigmatism. The two foci for paraxial rays are at $F_1$ and $F_2$, the astigmatic difference being the line $F_1F_2$.

Considering the two sections of the wave-surfaces separately, it is clear that they are identical with the sections of wave-surfaces produced by spherical refracting surfaces. Looking at each wave surface as a whole, it has different vertical and

horizontal or principal curvatures. The first saucer type of wave-surface is convex towards the incident light, the curvature of the vertical section being less than that of the horizontal section. It will be noticed that the left and right edges of the saucer (looking along the rays in the direction of the light) begin to turn in before the top and bottom edges so that the saucer type of wave-surface in the vertical sections is associated, for some distance, with the saucer with inturned edges type in the horizontal sections. Similarly the horizontal sections reach the closed surface type before the vertical sections. The wave-surfaces are never completely closed as is the case when the refracting surface is spherical. Again the horizontal sections

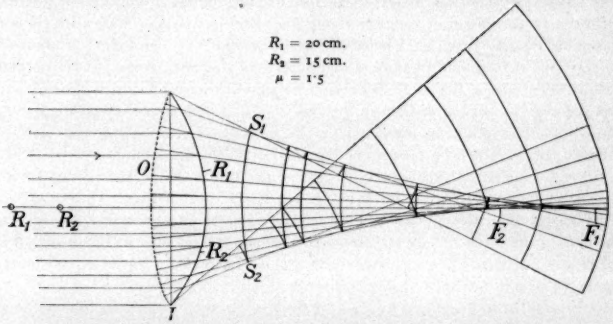



Fig. 7. Wave-surface of refraction. Plane wave, toric refracting surface, glass-air.

reach the goblet type before the vertical sections, and the basin type before the vertical section has finished with the goblet type. The focal lines are formed by those double folds and triple layers of the wave-surfaces which we have spoken of as the bases of the goblets. If we take a vertical section of the wave-surfaces through the focus $F_2$ at right angles to the plane of the paper, that is to say, if we imagine a screen placed at $F_2$ and arranged at right angles to the axis of the lens we shall see a bright vertical line limited in extent by the two extreme top and bottom rays in the plane of the diagram and surrounded by a less luminous area due to spherical aberration. If we put the screen at the focus $F_1$, we shall see a much longer bright horizontal focal line surrounded as before by a less luminous area, the line in this instance being limited in length by the extreme right and left rays as seen when looking along the axis in the direction of light. Between the two foci $F_1$ and $F_2$ the horizontal sections of the wave-surfaces are of the basin type and the vertical sections are of the goblet type. Beyond $F_1$ both sections are of the basin type, the vertical section having less curvature than the horizontal. It is usually said that the wave-surface between the two foci is of saddle-back form. This is only partially true. The base of the goblet type of wave-surface in the vertical sections has the central part convex towards the incident light whilst the

basin type in the horizontal sections is concave towards the incident light; but the greater parts of the wave-surfaces of both types, that is to say, the bowls of the goblets and the whole of the basins, are of the same curvature, both being concave towards the incident light.

## Some Properties of the Wave-Surface.

### *Differences of Phase.*

The difference of phase between disturbances propagated along any particular refracted ray and along the axis, at the point where the refracted ray cuts the axis, is represented on the diagrams by the axial length of the "closed surface" wave-surface, or by the axial length of the base of the goblet type of wave-surface. Reference to Fig. 1 will show that the optical length along the extreme marginal ray from the orthogonal surface to the axis is the same as the optical length along the axis from the orthogonal surface to the point where the central part of the wave-surface cuts the axis. The optical length of the base of the goblet divided by $\lambda$, the wave length, therefore gives the difference of phase. For example, the difference of phase between the disturbance propagated along the 4th refracted ray from the top in Fig. 1 and that propagated along the axis is represented by the axial length of the base of the goblet marked $G_1$.

Prof. Conrady* has given the following rigorous trigonometrical expression for this difference of phase

$$W = \frac{\mu' h \sin\phi' \sin\frac{\beta-\beta'}{2} \sin\frac{\phi-\beta'}{2}}{2\cos\frac{\alpha}{2}\cos\frac{\beta}{2}\cos\frac{\beta'}{2}\cos\frac{\phi}{2}\cos\frac{\phi'}{2}},$$

where $\mu$, $\mu'$ are the indices of refraction of two media separated by the refracting surface, $\phi$ and $\phi'$ are the angles of incidence and refraction, and $\alpha$, $\beta$, and $\beta'$ are the angles which the normal, the incident, and the refracted rays make with the axis.

For parallel incident paraxial rays this reduces to

$$W = -\tfrac{1}{8}\mu^2(\mu-1)\frac{h^4}{r^3},$$

where $h$ is the semi-aperture.

### *Diffraction spectra.*

The diffraction spectra will be found at the outer edges of the saucer wave-surfaces, at the inner edges of the flanges of the inturned edge saucer type, at the apex of the closed surface, and at the rim of the bowl in the goblet and basin types of wave-surface. In an uncorrected lens the only point in which diffraction rings will be seen at the centre of the field is at the focus for the extreme marginal rays. If the screen be moved further towards the paraxial focus the effect of the spherical aberration will be to move the diffraction rings further outwards from the axis.

* *Monthly Notices, R.A.S.* 65 (1904–05) 594.

*Interference Rings and Patterns.*

Interference patterns will be found in the area bounded by the two extreme marginal rays and the caustic. If we draw a number of inturned edge saucer type of wave-surfaces half a wave length apart, as shown in Fig. 8, in which alternate surfaces are drawn in dotted lines, we shall see that where the full lines intersect each other we shall get maximum intensity and that where a full line intersects a dotted line we shall get minimum intensity. By joining up points of maximum intensity we get lines of maximum intensity, as shown in Fig. 8; intermediate between these lines will be lines of minimum intensity not shown. By considering Fig. 8 to rotate round the axis we see that we shall get a series of surfaces of revolution, each of which is a locus of maximum or minimum intensity. A screen

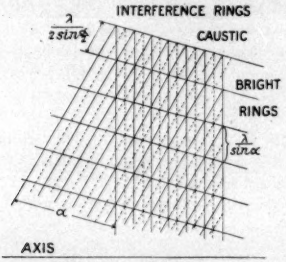



Fig. 8

placed at right angles to the axis will exhibit circular interference rings of maximum and minimum intensity. If the incident pencil be oblique, as in Fig. 6, a screen at right angles to the axis will show patterns which are oval in form and these will be the nucleus of the comatic flare.

If $\alpha$ be the angle between the two branches of the wave-surface at a crossing point, it is easy to see that the perpendicular distance between the bright lines in Fig. 8 is $\lambda/2 \sin \alpha/2$ and that the difference of the radii of successive bright rings is $\lambda/\sin \alpha$ where $\lambda$ is the wave length. In the neighbourhood of the caustic, where $\alpha$ is very small, the radial distance apart of the rings will be many wave lengths and the rings ought to be sufficiently far apart to be observed or photographed. Remote from the caustic the rings would be closer together and the difference between the intensity of bright and dark rings would be less, since both distance apart and amplitude of disturbance decrease as $\alpha$ increases.

Interference fringes similar to those just described occur in the case of the reflection of light by a concave mirror. Prof. Wood in his *Physical Optics* says "These fringes are easily seen by concentrating sunlight upon a pin hole and reflecting the light from an oblique concave mirror. The fringes will be found in the region between the primary and secondary focal lines, i.e. along the caustic."

In the discussion on Mrs Griffiths' paper referred to above, Commander Baker asked whether "any attempt had been made to disentangle the two effects," that is to say, the diffraction rings and the interference rings. I should like to suggest to Mrs Griffiths or Dr Martin that a simple plano-convex lens should be used with plenty of spherical aberration and that two photographs should be taken, one as nearly as possible at the focus for marginal rays and the other close to the focus for paraxial rays. The first should show the combination of diffraction and interference and the second should show interference alone since the diffraction rings would be shifted outwards towards the edges of the field.

*Characteristic of the Focus.*

It is clear from what has been said above that the characteristic of the focus, always present, whether the system consists of one or more refracting surfaces or whether it is corrected or uncorrected for spherical aberration, is what may be described as the "double fold and triple layer" of the wave-surface forming what has been called the base of the goblet.

If the axial length of the base of the goblet be a quarter of a wave length, it will be seen from Fig. 9 that successive wave-surfaces half a wave length apart will not intersect at all, so that from the point on the axis where the difference of phase is $\lambda/4$ up to the focus for paraxial rays the rotation round the axis of the caustic and

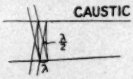



Fig. 9

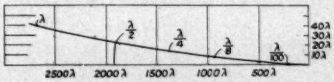



Fig. 10

the central part of the wave-surface will generate a focal volume in which there will be no destructive interference. By destructive interference I mean that successive wave-surfaces half a wave length apart will not intersect. This might be called the "*Focal Volume*" and its axial length might be called the "*Depth of Focus*." We should thus give a precise definition to the present rather vague term "depth of focus."

In the presence of sufficient spherical aberration to shift the diffraction rings well away from the centre of the field a screen placed at right angles to the axis and cutting the focal volume would show a central bright disc surrounded by a less bright area. The three layers of the wave-surface would make the illumination of the central disc three times as intense as that of the surrounding area. Fig. 10 shows the caustic plotted on a large scale, the divisions along the axis of $y$ representing 10 wave lengths and those along the axis of $x$ 500 wave lengths, a wave length being taken as 5000 tenth metres. The points on the caustic marked $\lambda$, $\lambda/2$, $\lambda/4$, $\lambda/8$, $\lambda/100$ are the points where the bases of the goblets touch the caustic, the axial lengths of the bases of the goblets at these points being $\lambda$, $\lambda/2$, etc. The part from $\lambda/4$ to 0 represents half the section of the focal volume. The central part of the wave-surface corresponding to a difference of phase of $\lambda/2$ is drawn, but I have

not drawn the rest of the base of the goblet because half a wave length would not show on the diagram as it would only be 1/1000 of the divisions along the axis of $x$. The following table gives the particulars of the points shown on Fig. 10.

Plano-convex lens $+5D$,
Power of 1st surface $=0$,
" 2nd " $=+5D$,
Radius of 2nd surface $=\cdot1$ metre,
Index of refraction $\mu=1\cdot5$,
Focal length $=\cdot2$ metre.

$\lambda=5000$ tenth metres,
$n=2000$ wave lengths to a millimetre,
$f-l=$distance of wave-surface from the paraxial focus when $h=0$,
$h=$semi-aperture,
$y_c=$ordinate of the caustic and of the wave-surface.

| $h$ | $W$ | $f-l$ | | $y_c$ | |
|---|---|---|---|---|---|
| mm. | | mm. | wave lengths | mm. | wave lengths |
| 7·72195 | $\lambda$ | 1·3443 | =2688 | ·019997 | 40 |
| 6·49336 | $\lambda/2$ | ·94975 | =1899 | ·011875 | 23·75 |
| 5·46024 | $\lambda/4$ | ·67137 | =1343 | ·007058 | 14 |
| 4·59149 | $\lambda/8$ | ·47486 | = 949·7 | ·004198 | 8·39 |
| 2·44189 | $\lambda/100$ | ·11409 | = 228·2 | ·000494 | ·09 |

$h$ was calculated from the formula

$$W=-\tfrac{1}{8}\mu^2(\mu-1)\frac{h^4}{r^3}.$$

$l$ (see Fig. 11) was computed from the equation to the wave-surface

$$x=\{l+\mu r(1-\cos\theta)\}\cos(\theta'-\theta)+r\cos\theta,$$
$$y=-\{l+\mu r(1-\cos\theta)\}\sin(\theta'-\theta)+r\sin\theta,$$

by putting $y=0$.

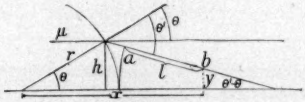



Fig. 11.

$y_c$ was calculated from the equation to the caustic

$$y=\mu^2 r\sin^3\theta,$$
$$x(\mu^2-1)=\mu r(\mu\cos^2\theta+\cos^2\theta'),$$

by putting $\sin\theta=\theta=\frac{h}{r}$. This equation may be written

$$y^2=-x^3\frac{8}{27}\left(\frac{\mu-1}{\mu}\right)^2\frac{1}{f},$$

where $x$ is measured from the focus for paraxial rays, that is to say, $x$ is what we have called $f-l$ in the table.

*Position of best Focus.*

We may consider the caustic curve to be a straight line within the limits of the focal volume and it follows that the area of the triple layer of the wave-surface is proportional to the square of its distance from the paraxial focus. Assuming that

there are $n$ rays of light passing through unit area of the wave-surface it follows that there are $3n\,a^2$ passing through any section of the wave-surface whose area is $a^2$. As the area $a^2$ is diminished in proportion to $b^2$ where $b$ is its distance from the paraxial focus, it follows that the number of rays passing through a section divided by the area of the section is a constant, that is to say, the intensity of illumination in the focal volume is a constant except very near the caustic. The focal volume can therefore be defined as a volume in which there is no destructive interference and in which the intensity of the light is a constant.

The position of best focus, if one wishes to have as much light as possible, is therefore just inside the focal volume where the phase difference is $\lambda/4$. If one wants the least area regardless of quantity of light, the best position is the nearest distance practicable to the paraxial focus. This is on the assumption that considerable spherical aberration is present.

## Conclusion.

In conclusion I hope my paper will stimulate the study of the wave-surface of refraction. The ray method has had a long innings and it is time that the wave method should yield up the results of which it is undoubtedly capable. The two methods ought not to be regarded as separate but as interdependent and each should be used in the manner for which it is most fitted, for after all the wave-surface is more real than a ray and should be looked upon as a shorthand note of a bundle of rays*.

## DISCUSSION.

Mr T. Smith: Any contribution which enables us to form a clearer mental picture of the events taking place as light advances through space is of obvious importance, and we are greatly indebted to Mr Whitwell for his most interesting treatment of the complex series of events which occur as light travels to a focus not free from aberrations. It is quite evident that Mr Whitwell has expended a great amount of labour in constructing his diagrams, and they will undoubtedly be found of value. The difficulty of preparing such diagrams of high accuracy is perhaps not so readily appreciated. In the case of highly corrected lenses the errors of phase must not exceed some fraction of a wave length, and, as Mr Whitwell has pointed out, the differences of path on their natural scale are given by certain dimensions of the wave-surface. It is not therefore surprising to find that the direct method pursued in the paper is only feasible where the aberrations reach large values. If it were intended to construct accurate diagrams of this kind corresponding to the performance of well corrected instruments another method must be substituted. One of the simplest consists in deriving the curves algebraically from a potential function, which can then be plotted with the scale of path differences magnified to

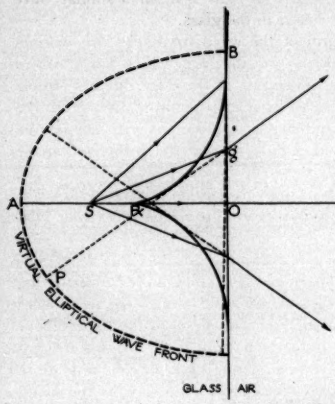
* I have to thank Mr Ernest Hooper of 62 A, Hill Road, Wimbledon, for his kindness in making my lantern slides. If anyone engaged in teaching would like a set of the slides, which includes a number of illustrations not reproduced in this paper, Mr Hooper has the negatives and would be pleased to supply sets at ordinary prices.

any desired extent, so that the characteristic changes are readily appreciated. The function may be selected by observing that it gives the proper relation between aberration and ray position. If this is specified by a curve connecting longitudinal aberration and aperture, the required function may be obtained by integration: if the shape of the caustic is given, a somewhat more complex process is involved; a knowledge of phase differences provides a direct solution. By one means or another it is possible to select a function from which the desired curves can be traced.

Any one of these alternative forms—potential function, aberration diagram, wave-surface, caustic—constitutes a theoretically complete basis for the determination of all the properties of the image. Some happen to be more easily constructed than others. Unfortunately, the one we should undoubtedly prefer to all alternatives, the intensity distribution at any point in terms of its co-ordinates, involves such an extraordinary amount of labour for its determination that for nearly all purposes a simpler alternative must be substituted. Some passages in Mr Whitwell's paper seem to imply that details of the image appearance can be inferred directly, that is, without mathematical treatment, from the wave-surfaces. If this view could be accepted the theory of images would be a very much simpler affair than in fact it is; some of the deductions made by the author on this basis are at variance with those found by accurate calculations on the wave theory. Whatever the original basis may be, there appears to be no short easy direct road leading to a point from which all the intricacies of the diffraction pattern in various planes are clearly appreciable.

Comdr. Baker: The paper shows in a manner that appeals at once to the eye the forms which the wave front may take up when approaching the focus and particularly it illustrates some of the types of wave form in the presence of varying degrees of aberration. The graphical construction that Mr Whitwell utilizes must be employed with extreme caution as very small errors in measurement will result in a distorted wave. For example, when looking at his large scale diagrams previous to the meeting I noticed in one or two places that his wave front as determined by measurement was not at right angles to the rays as determined by calculation, the departure from the right angle being quite apparent to the eye. In one or two simple cases the correctness of the drawing can be checked, but the general case of a spherical wave being refracted at a spherical surface gives rise to refracted waves of troublesome form.

In the case Mr Whitwell illustrates in Fig. 4, where a spherical wave is refracted through a plane surface, one of the wave fronts is an ellipsoid or hyperboloid of revolution. If the source of light is in the glass at a distance $C$ (Fig. A) from the centre then a virtual refracted wave front is formed by the revolution of an ellipse of semi-axes $Cn$ and $C\sqrt{n^2 - 1}$. The caustic is the four-cusped evolute of the ellipse having one cusp $R$ at the image point corresponding to $S$ and two cusps upon the glass surface at the points at which a ray strikes at the critical angle. The ellipse can be graphically constructed with great accuracy and it would be interesting to see how Mr Whitwell's construction for rays and for wave front would stand up




S is point source of light SO = C
R image point RO = C/n
Semi major axis AO = Cn
Semi minor axis BO = $C\sqrt{n^2-1}$
Eccentricity e = 1/n
S is one focus of the ellipse

In any ellipse Sg = e Pg
Hence time in glass from S to g
is equal to time in air from P to g

Fig. A

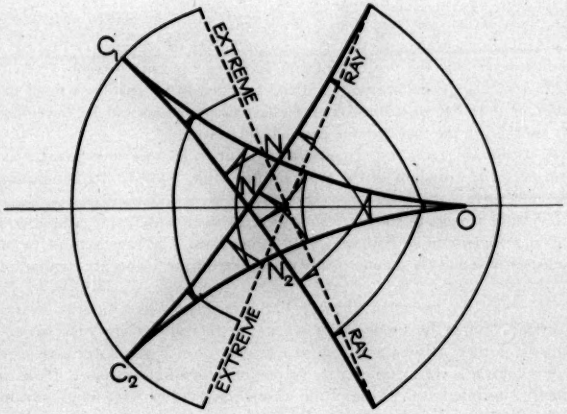



Fig. B

to a test of this nature. When the point source of light is in air a similar wave front, but this time a hyperboloid, can be found in the glass.

Mr Whitwell's classification of the form of the wave front into five distinct types is insufficient when the caustic departs from the form which has a simple axial cusp. A lens corrected for spherical aberration at the edge offers a simple illustration. The caustic in this example has an axial cusp pointing away from the lens and as the ray is inclined at steeper and steeper inclinations with the axis its point of contact will ultimately reach another cusp so that the complete caustic must contain at least three cusps. Naturally these three cusps are very close to one another and it would be extremely difficult to draw a diagram accurately to scale. Professor Filon and I gave an example of such a caustic in our paper (*Trans. Opt. Soc.* 20 (1918–19) 67).

It should be noticed that once the caustic is drawn there is no difficulty in constructing the wave fronts from it by the ordinary mechanical method of constructing involutes of a given curve. For instance, suppose we wish to draw the wave front passing through $N_1$ and $N_2$ in the illustration, Fig. 1 on p. 69 of the paper above referred to. Attach a string to the cusp $O$ and stretch it out to $N_1$ not in a straight line but along the caustic. This can be done by constructing a convex template to fit $ON_1C_1$ and making the string lie along the convex edge. Now unwrap the string keeping it always taut. The end of the string as it travels away from $N_1$ will trace out the wave front. The point of contact of the string (i.e. of the ray of light) will travel along the caustic to $O$ when the string will lie along the axis. After this position the string will begin to wrap itself along $ON_2$ until finally the end of the string reaches $N_2$. In this way the wave front from $N_1$ to $N_2$ can be formed. Now attach the string to $C_1$ and make its length equal to $C_1N_1$ measured along the caustic. Repeat the same method of describing the wave front and so fill in the part between $ON_1C_1$ and $N_2NC_1$. A similar procedure with a string attached to $C_2$ enables a third part of the wave front to be constructed and the last two portions can be developed by strings attached to the two extreme ends of the caustic.

Even if this process is not actually carried out on the drawing-board a mental performance of it will allow of the general form of the wave fronts being pictured without difficulty.

A few wave fronts are shown (Fig. B) for a lens in which spherical aberration goes up to a maximum and subsequently diminishes. The caustic must be of the three-cusped type and the geometrically constructed wave fronts are easy to follow.

Mr Whitwell: I quite agree with Mr T. Smith that the graphic way of drawing wave-surfaces cannot be used except when the aberrations are very large. My drawings were merely intended to give a general idea of the successive forms a plane or spherical wave assumes after refraction at a single surface. I was more particularly interested in the form of the wave-surface a few wave lengths from the focus of a single refracting surface and as the graphic method could not be used I was compelled to plot the results of calculations to illustrate the form of the wave-surface in this case.

Assuming that the disturbance constituting light is in the same phase at every point of a wave-surface and assuming also that Huygens' construction gives a true wave-surface, I do not see why it is not possible to deduce some properties of an imaging pencil of rays directly from the forms of the wave-surfaces without mathematical treatment as Mr Smith implies; for example, Prof. Wood appears to have deduced in this way, and verified by experiment, the presence of interference fringes in light reflected obliquely from a concave mirror.

In reply to Commander Baker, I was familiar with the fact that the section of the isotomic surface in the particular case of his Fig. A is an ellipse but did not use this property in drawing my Fig. 4 as I considered that the method used in the paper involved less trouble. It is of course easy to prove that one of the loci given by the construction I used is, in this particular case, an ellipse. As regards checking the accuracy of the drawings by utilizing this property it seems to me simply a point of skill in drawing, since both methods must give the same result. Perhaps I have not made it sufficiently clear in the paper that my construction is merely a simple way of drawing one of the envelopes of a series of circles; it is in fact Huygens' construction.

I agree that the classification of wave-surfaces in five types is insufficient to cover every case, but I think that in every case these five types will be found but with additions; thus, in Commander Baker's Fig. B, if the aperture be cut down so as to exclude all the rays above $C_1$ and below $C_2$, it will be seen that all the wave-surfaces belong to one of the five types; the marginal bundles of rays will have the effect of adding portions on to the outer edges of the five types formed by the central rays. The effect seems to be that we jump from the saucer type to a saucer combined with two goblets. I was much interested in this Fig. B as Commander Baker shows, so easily, the forms a wave assumes after passing through a corrected lens.

# STEREOSCOPY RE-STATED

By JAMES WEIR FRENCH, D.Sc.

*MS. received*, 11*th April*, 1923. *Read and discussed*, 10*th May*, 1923

*ABSTRACT.* While the phenomenon of stereoscopy has been to a limited extent developed in a practical manner, there appears to have been but little actual research work done within recent years, the impression being a general one that the particular field had been already thoroughly explored.

If an attempt be made to reproduce the experiments described by Wheatstone in his original paper, in which his great discovery of stereoscopy was published, it will be found that the results are somewhat different in important respects from those described.

From experiments recently carried out by the author, it appears that stereoscopic vision is possible only within certain limits. For certain pairs of objects, whether on the same or different horizons, there are generally two extreme critical points beyond which stereoscopic vision breaks down. For certain pairs of objects on the same horizon there are two inner critical points. Objects of dissimilar form but approximately the same average angular dimensions can frequently be combined stereoscopically: thus a circle can be combined with a triangle if their average angular dimensions are about equal. When the angular dimensions are very different, combination is generally impossible: thus, for example, a thin line cannot be combined with a thick line or triangle or other form of considerably larger size. When the pairs of objects are dissimilar in size there is only one pair of outer and inner critical points if the objects are on the same horizon and one outer critical point if they are on different horizons. For pairs of objects any of which can be combined together, there are two pairs of such critical points.

Data regarding the positions of the critical points under various conditions are contained in the paper, which also deals generally with the effects of variation of the conditions of observation and particularly variations of the illumination.

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Sir David Brewster* in his work on the stereoscope has remarked that "Optical writers who lived after the time of Aguilonius† seem to have considered the subject of binocular vision as exhausted in his admirable work." That the subject of stereoscopic vision has been exhausted is a view that has been quite recently expressed by others.

This impression, which appears indeed to be one of too general acceptation, has discouraged the continued research that so interesting a subject merits. In order to convince ourselves that the field is a large and profitable one, it is only necessary to reflect how deficient is our present-day knowledge of the retinal and cerebral mechanism involved in vision and particularly in the appreciation of solidity.

* *The Stereoscope*, 1856.
† *Aguilonius Opticorum.*

Many who are well informed in most branches of optical science have been content to accept the explanations of stereoscopic vision offered by Wheatstone, the discoverer, and Brewster, the principal pioneer investigator of that most remarkable phenomenon, synthetic solidity. There are many who still regard stereoscopic vision as a function of binocular convergence and who are surprised to learn that the convergence of the eyes is a factor of but secondary importance.

In early Victorian days, when science was an approved subject of parlour conversation, the popularity of the stereoscope reached its zenith, but the expectations of its development and utility, as expressed by Sir David Brewster, have not been realized. To-day the subject is a comparatively dead one in contrast with its earlier activity.

Its application of greatest monetary importance is probably that of Grousillier to the measurement of ranges.

In the stereo-photo-theodolite and the stereo-microscope the principle has found a special but not very extensive use.

Notwithstanding its hitherto somewhat limited application, stereoscopy from the scientific point of view is, I think, nevertheless well worthy of consideration, and particularly of research, for theory unsupported by experiment is of but little value for the development of such a subject.

Binocular vision confers upon its fortunate possessor certain benefits, the respective values of which cannot, however, be readily assessed. While one eye is undoubtedly less useful than two, the disability of total blindness is incomparably more serious. After the lapse of such time as is necessary for a natural readjustment to the new conditions, the visual power of a one-eyed person is not so seriously limited as might be expected.

Under ordinary photoptic conditions, that is, the condition under which the greater part of our active life is spent, binocular vision does not confer the advantage of double illumination. That the difference is small can be readily tested by observing with one eye and then two, but in any such experiments it is necessary to close the eye only for periods sufficiently short to prevent it when closed from becoming dark-adapted to any appreciable extent.

It should also be remembered that the photometric values of the two eyes may differ by appreciable amounts.

As a spare part, the second eye may be of the greatest value, but unfortunately this advantage is often lost when the affection of the eye is of a serious nature, as in such a case the other eye frequently becomes involved.

As regards the extension of the field of view sideways by about 30° on the previously blind side, the gain attributable to the second eye is certainly of importance, however indistinct the vision over the region in question may be.

That the limitation of the field of view is a serious one was realised by Augustus de Morgan* who suffered from a grievous infirmity—the loss of one of his eyes—the unfortunate result of a childhood too long spent in India. At school we are told his loss provoked all kinds of gibes and practical jokes amongst the boys and

* *Memoirs of Augustus de Morgan*, 1882, p. 5.

it was the practice of one, until he was trapped and thrashed, to approach de Morgan on the blind side, hold a sharp pointed penknife to his cheek, and call him suddenly by name, with painful results to the victim.

While two eyes may not often be necessary as a guard against such thoughtless tricks, the extension of the binocular field is invaluable in these days of rapid and congested traffic.

Binocular vision sometimes confers the benefit of stereoscopy—that is, the power of appreciating the appearance of apparent solidity—which is the subject of this paper. While it is possible monocularly to discriminate differences of distance and thus to appreciate solidity as the result of parallactic movement, light and shade, and experience, there is this essential difference between stereoscopy and perspective, that for the appreciation of the former two separated eyes are essential, while for the latter one suffices.

Given the faculty, it is the separation of the eyes that primarily determines quantitatively the degree of stereoscopy. As the equivalent separation is decreased by the use of artificial aids, the power diminishes until a zero value is reached which corresponds with monocular vision. Since it is the linear separation or ocular base length that is the determining factor in stereoscopy, it follows that the appearance of solidity is determined by the appreciation of dimensions measured in directions parallel to the ocular base.

Two series of vertical lines drawn on a plane surface may be so spaced that they appear stereoscopically, that is, at different distances, when viewed naturally. If the lines are turned from the vertical position into the horizontal the stereoscopic solidity disappears, owing to the vertical ocular base being zero, and the lines then appear to lie upon one surface, as is actually the case.

For the appreciation of solidity it is necessary that the vision should be at least fairly distinct and the region in which the appearance is recognizable is determined principally by the space that can be viewed simultaneously in direct vision by the two eyes, by their resolving power, and by the lower limit of accommodation, namely the distance of the punctum proximum.

Stereoscopic vision is not an inevitable consequence of the possession of two eyes. There are many who lack the power entirely and many who possess it to a more or less limited extent. The number of good stereoscopic observers is not great and in the development of their faculty careful training is often necessary. There appears, just as in the resolution of defects of alignment, to be no relationship between the separation of the macular cones and the stereoscopic resolving power of the eye, there being skilled observers who under ideal conditions can appreciate a stereoscopic parallax angle of four seconds, which is much less than the angle subtended by any fully developed retinal cone. But it must not be assumed that such results are regularly or consistently obtained. Under normal natural conditions the uncertainty of coincidence parallax observation of the unaided eye is usually assumed to be twelve seconds. It will be assumed that the stereoscopic resolving power of the unaided eye is twenty seconds of angle, although so small a value is not easily obtained. If the separation of the eyes is 63 mm.,

then, since $\theta = B/R$, where $\theta$ is the parallax of an object at a distance $R$ in metres and $B$ is the separation of the eyes, namely 63 mm., it follows that

$$\frac{20}{200{,}000} = \frac{63}{1000 \times R}$$

and

$$R = 630 \text{ metres.}$$

At this comparatively short distance the benefit of stereoscopic vision is entirely lost. Beyond it the natural ocular base length affords no assistance in the determination of the relative distances of objects or the appearance of solidity, because with a base length of 63 mm. the eyes cannot discriminate the separation of an object at infinity from one at about 630 metres' distance.

For stereoscopic vision the punctum remotum of the unaided eye is accordingly about 600 metres and the punctum proximum is that of ordinary near distinct vision, namely about 25 cm. An extension of the stereoscopic radius of vision can be obtained by an artificial increase of the base or an increase of the magnification. Thus a base length of 2 metres and a magnification of 20 diameters would together increase the radius from 600 metres to 400 kilometres and it is by such means that the accurate stereoscopic measurements are obtained.

Having thus briefly indicated the nature and limitations of stereoscopic vision, it may be of interest to consider the historical basis of our knowledge of this remarkable phenomenon, and in doing so I shall describe certain experiments of my own that appear to have disclosed four critical points in stereoscopic vision. That stereoscopic vision failed when two suitable stereoscopic images were brought close to each other side by side was generally known, but it does not appear to have been realized hitherto that there are under certain conditions four critical points and that the existence of these points is an inevitable consequence of the theory of stereoscopic vision which, when understood, enables the very variable positions of the critical points to be predicted.

Historical ground is always uncertain territory. There can be little doubt, however, that at a very early date it was realized that a solid object generally presents different aspects to the two eyes of an observer. As to whether or not it was realized that the images of the object impressed upon the retinae were different from each other and that the appreciation of solidity necessitated the composition of dissimilar images by the visual sense, the seat of which was uncertain, is another question. In the early treatise on Optics published in Euclid's name in 1557 three of the theorems, which must have been elaborated over 2000 years ago, clearly expound what has been described by Sir David Brewster as a palpable truth; but in the use of this adjective Sir David may not have been entirely disinterested, as his anxiety to discount certain claims of Wheatstone is very evident in his writings.

François d'Aguillon, a Jesuit of Brussels, in his comprehensive "Optical Treatise" published at Antwerp in 1613, describes in detail the impressions received by each eye, and in this work there is used for the first time the expression stereographic projection.

Gassendus*, Baptista Porta†, and others overcame all difficulties of binocular vision to their own satisfaction by assuming that the eyes were used alternately, not simultaneously. But previously, in the year 1550, that most remarkable of Greek physicians, Galen, had expressed the clearest of views regarding binocular vision. Sir David Brewster, after quoting the statement of Galen‡, remarks:

> In such distinct and unambiguous terms, intelligible to the meanest capacity, does this illustrious writer announce the fundamental law of binocular vision—the grand principle of stereoscopy, namely "that the picture of the solid column which we see with both eyes is composed of two dissimilar pictures as seen by each eye separately."

The words "intelligible to the meanest capacity" are a reflection, let us hope unintentional, upon the intelligence and capacity of Wheatstone who unfortunately, although he appears to have been acquainted with Galen's work, did not recognize his statement regarding binocular vision and indeed appears to have desired to deny him the credit of its unambiguous representation.

Acceptance of the principle of simultaneous binocular vision of a single object led naturally to the consideration of what, since the time of Alhazen, nearly 1000 years ago, have been termed corresponding points, or occasionally corresponding regions.

Helmholtz and other German writers sometimes use the term "identical points" —a term that was first introduced by Professor Johannes Müller, but which is hardly a correct expression, as two separated points may more properly be described as corresponding rather than identical.

Alexandre Prévost in his essay on "The Theory of Binocular Vision" is responsible for the definition of which the following is a translation:

> Two points situated one upon the right and the other upon the left retina and such that when they receive rays from the same luminous point they transmit to the mind a single sensation are called corresponding points of the two retinae.

The points of fixation, that is, the centres of the two foveae, are corresponding points. Other corresponding points are similarly placed on the two retinae, not symmetrically, that is, they lie on the same sides of the fixation points, i.e. either both to the left or both to the right, but not necessarily at equal distances.

Prévost's definition is a convenient but not complete one. Suppose, for example, in Fig. 1 that $a$ and $b$ are the two eyes, $c$ and $d$ the mean nodal points which may be assumed to coincide with the centres of rotation of the eyes, and $E$ and $F$ two luminous object points. If $E$ is the fixation point, that is, if the eyes are directed upon $E$, in which case the optical axes intersect at $E$, then images of $E$ will be formed upon the foveae of the eyes at $g$ and $h$ respectively; as the foveae are corresponding points a single image $e$ of the object $E$ will be appreciated by the brain. If the object point $F$ is so situated that its images fall upon corresponding points $k$ and $l$ a single image $f$ of the object point $F$ will be appreciated by the brain. Each object point

* Gassendus, *Opera*, 1568.
† Joan. Baptista Porta, *De Refractione*, Book VI. Naples, 1593.
‡ *De Usu Partium Corporis Humani*, 1550.

will create a single image impression on the brain in accordance with Prévost's definition.

But Wheatstone has shown us that the use of single luminous points as stated by Prévost is not an essential feature of a definition of corresponding points. For the single points $E$ and $F$ there may be substituted separated pairs of luminous points $E$ and $E'$ and $F$ and $F'$ mounted, it may be, in the focal planes of collimators $M$ and $N$, as indicated in Fig. 2. These collimators, when suitably adjusted, may be disposed with their optical axes parallel or inclined without affecting the visual appearance. A result identical with that of Prévost is obtained and this particular illustration is of interest as an indication of what will be shown later, namely that stereoscopic vision is not determined by the inclination of the optical axes.

Both illustrations demonstrate what has already been stated, that corresponding points such as $k$ and $l$ are both situated on the same side, either temporal or nasal,

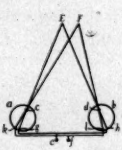


Fig. 1

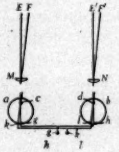


Fig. 2

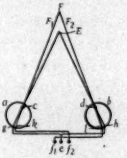


Fig. 3

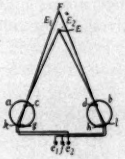


Fig. 4

of the axial points $g$ and $h$ which also are corresponding points, and are indeed the foveae of the retinae.

Similar obliquity of vision of the two eyes is in such cases a feature of the appearance of a stereoscopic image point relatively to the point of fixation; that is to say, the second object must be viewed from the one side or the other of the optical axes.

Consider, for example, the case illustrated in Fig. 3, in which the two objects lie in the median plane.

Rays from the fixation point $E$ will fall on the foveae $g$ and $h$ and form a single image $e$. Rays from $F$ will fall on the disparate points $k$ and $l$ and form two mental images $f_1$ and $f_2$ at the positions $F_1$ and $F_2$. If $F$ is fixed instead of $E$, a somewhat similar result is obtained, as indicated in Fig. 4. In neither of these two cases is the vision stereoscopic: the two objects appear as three, one of them being doubled. But with practice it is possible to fix a position intermediate between $E$ and $F$, as in Fig. 5, where $g$ and $h$ are the foveae on the axes which intersect at the intermediate fixation point. Now, if the sources $E$ and $F$ are sufficiently alike—and that is a supposition of importance—the excitation of the corresponding points $f_1$ and $e_2$ will produce a single mental image to which the sources $E$ and $F$ contribute. Similarly $e_1$ and $f_2$ which both lie on the right hand sides of the optical axes will form a single image to which both sources contribute.

These illustrations will serve to indicate how incomplete is Prévost's definition,

how difficult it is to generalize the subject of stereoscopy, and how important it is to consider in any particular experiment what is the point of fixation of the eyes.

From what has been said it may be concluded that corresponding points are situated on horizontal sections of the retina parallel to the ocular base line joining the centres of the two eyes. In Fig. 6 the circles *A* and *B* represent front views of the retinae of a pair of ideal eyes, the foveae of which are marked *c* and *d*. These foveae are corresponding points. The line *cd* is the ocular base and may be regarded in the ideal eye as the principal horizontal meridian of the retina. Lines *kl* and *mn* drawn through the foveae are the principal vertical meridians. Points *e* and *f* on the same side of the vertical meridians are corresponding points and when excited produce a single mental image. Similarly *h* and *g* are corresponding points, but *f* and *g*, lying on opposite sides of the vertical meridians, are disparate points and when excited produce two distinct images. Points *r* and *s* on the horizontal meridian *pq* parallel to the ocular base are similarly corresponding points and give rise to a single image, but the brain cannot combine a point image such as *e* with one on the corresponding side but on a different horizon, such as *t*.

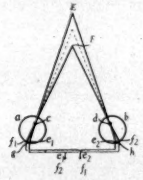


Fig. 5

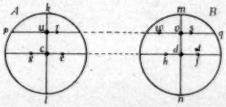
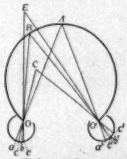


Fig. 6



(Prévost's Fig. 5.)

Fig. 7

Prior to the time of Wheatstone it was thought that corresponding points existed in definite pairs. Thus corresponding with *e* on the one retina there could only be one point *f* on the other retina at an equal distance from the centre, and similarly the point *s* alone could correspond with *r*. An image resulting from the excitation of *e* and *f* would lie on the horopter corresponding with the image of *c* and *d*. Wheatstone's great discovery showed what the admirers of the horopter were reluctant to admit, that any number of points similarly situated on the same horizontal meridian could be found to correspond with the point *e* and that, whereas points equally distant from the centre such as *e* and *f* produced an image on the horopter of *c* and *d*, all other pairs of points at unequal distances produced images situated at different distances in front of or behind the horopter.

The special case of a pair of points such as *u* and *v* on the vertical meridian is worthy of consideration. They form a single image. But *u* and any point such as *s* will also form a single image, and similarly *u* and any point such as *w* on the opposite side from *s* will form a single image, the one being on the far side of the *uv* horopter, and the other on the near side. When one of the points lies on the vertical meridian, either point *w* or *s* may be then regarded as being similarly situated. But the problem is still further complicated by the fact that oblique stereoscopy is possible and that

a vertical meridian does not necessarily pass through the fovea centralis as indicated in the diagram.

The idea of the horopter was introduced by Aguilon, who considered what would be the locus of points in the object space which cause single vision, on the assumption that the single vision of an object results from the excitation of corresponding retinal points. He concluded that all objects seen at the same glance with both eyes appear to lie in one plane containing the horopter, which he defined as being a line drawn through the point of intersection of the optic axes, that is, the point of fixation, and parallel to the line joining the centres of the two eyes. The plane of the horopter containing this line is also normal to that containing the optic axes. The words "at the same glance" indicate that for a particular horopter there is a particular condition of convergence and accommodation of the eyes. But at any considerable distance a large change of range may involve a quite inappreciable change of accommodation and convergence and, therefore, instead of being a line, the horopter is a horizontal plane of considerable extension in the direction of the line of sight.

At long ranges the horopter on this account has but little significance. At short ranges the horizontal horopter is not the simple straight line parallel to the ocular base defined by Aguilon. Johannes Müller* defined the horizontal horopter as a circle passing through the point of fixation and the centres or mean nodal points of the two eyes.

In Fig. 7, which is reproduced from Prévost's "Essay on Binocular Vision†," if $A$ is the fixation point and $O$ and $O'$ the centres of the eyes, then the circle $OAO'$ is the horopter. An object $B$ on this circle will excite the corresponding points $b$ and $b'$ of the retinae equally distant from the axial points and produce a single cerebral sensation of $B$. Until the contrary was demonstrated by Wheatstone, it was thought that objects $E$ or $C$ not lying on this horopter would excite what were regarded as disparate points of the retinae, and would appear duplicated. If the eyes were fixed on $E$ the horopter would be a circle of larger radius passing through $E$ and, as before, the centres of the eyes.

According to Prévost, the vertical horopters were lines perpendicular to the plane of the optic axes and passing through the points of fixation. It is unfortunate that these horopter diagrams are generally drawn out of all proportion. They convey an erroneous impression of the actual circumstances and, except as an interesting mathematical proposition, they are of very little practical value, as they are based on numerous assumptions that are known to be doubtful.

In Helmholtz's *Collected Scientific Papers*, vol. II, there are three papers dealing with the horopter and its form, which is of a much more complex character than the simple circle of Müller when physiological conditions more in accord with actuality are taken into consideration. Helmholtz's treatment of the subject is elaborate and the problem appears to have attracted him mainly as an interesting mathematical proposition.

* Johannes Müller, *Handbuch der Physiol. des Menschen*, 1840.

† Alexandre P. Prévost, *Essai sur la Théorie de la Vision Binoculaire*, Geneva, 1843 (15th Dec. 1842).

He appears to have entertained some doubts as to the value of these particular mathematical investigations, as in the second paper* he states

> The question may be asked as to whether or not to seek the exact form of the horopter is only a theoretical amusement, as Recklinghausen† has stated. I must confess (he continues) that for long I have been of the same mind. Recently, however, I have convinced myself that the appreciation of those object points that lie in the horopter has certain advantages that give the horopter a practical meaning.

He does not, however, attempt to disclose the advantages claimed in justification of his labours.

The horopter has been referred to, not because of any value it may have as an optical proposition, but because around the subject is centered a great deal of the early opposition to the claims of Wheatstone, the discoverer of stereoscopy. It was claimed by Wheatstone that the principle of the horopter was fundamentally false, as in stereoscopic vision the excitation of similarly situated but disparate points by an object not lying on the horopter may give rise to a simple sensation.

So revolutionary an assertion could not readily be accepted by the upholders of the horopter and much ink was spilled by the combatants.

Immediately prior to Wheatstone's great discovery it was generally accepted that in binocular vision:

(1) Both eyes were used simultaneously, not alternately.

(2) The appreciation of the object form was a resultant effect of two images impressed one upon each retina, and it was recognized that the two images were dissimilar in form.

(3) Simple vision resulted from the excitation of corresponding points of the two retinae.

(4) The excitation of disparate points necessarily resulted in double vision.

(5) Object points which appeared single at one glance of the eyes were situated upon the horopter.

(6) Object points not lying upon the horopter corresponding with a particular convergence and accommodation necessarily appeared doubled.

Wheatstone's discovery made it necessary to abandon items (4) and (6), which many were extremely reluctant to do.

In the *Philosophical Transactions* of 1838, Sir Charles Wheatstone published his "Contributions to the Physiology of Vision—Part the First—On some Remarkable and Hitherto Unobserved Phenomena of Binocular Vision." He commences paragraph 2 with the statement that

> It being thus established that the mind perceives an object of three dimensions by means of the two dissimilar pictures projected by it on the two retinae, the following question occurs: what would be the visual effect of simultaneously presenting to each eye instead of the object itself its projection on a plane surface as it appears to that eye?

Wheatstone recognized that a three-dimensional object formed two dissimilar two-dimensional images on the retinae and that the same retinal images could be

* "Ueber dem Horopter." Helmholtz, *Collected Papers*, vol. II, p. 452.

† *Archiv für Ophthalmologie*, vol. V, pt. II, p. 146.

formed on the respective retinae by the observation of appropriate two-dimensional objects. Might it not, therefore, be possible, he argued, to produce synthetically the appearance of a three-dimensional object by the observation of two-dimensional pictures?

In propounding the question he did what no one before him had done and in the final result he demonstrated once more his power of invention based on well directed deduction.

The next step—the formation of the requisite images—was an easy one and the various arrangements of mirrors adopted by him are so well known that they need not be described here.

It was generally accepted that the similar excitation of corresponding points of the two retinae gave rise to the appreciation of a single image. Wheatstone extended this idea by demonstrating that the excitation of disparate points of the two retinae may give rise to the appreciation of single images whose distances depend upon the amounts of the disparities.

Reference has already been made to the opposition that was aroused by such a claim and a careful survey of Wheatstone's work does disclose a weakness of description and a want of historical knowledge that afforded some ground for criticism.

When repeating his experiments it is difficult to understand his descriptions of the results to be expected, but in a statement of this kind there is no desire to detract from the greatness of Wheatstone's discovery which is unassailable.

There is a fundamental consideration underlying all binocular vision experiments to which Wheatstone makes no reference and which does not seem to have been recognized by him or indeed sufficiently realized by subsequent writers. It is this: that for the unaided eye accommodation and convergence are closely, although not rigidly, associated. Normally a particular condition of accommodation occasions a particular convergence, and to alter the convergence independently of the accommodation involves the exercise of mental effort or the use of artificial aids.

For the correct interpretation of the phenomena of binocular vision, it is essential that this important factor should be realized, as otherwise error in description may readily occur, even though the final result itself may be correctly expressed. A simple experiment will serve as an illustration.

If two stereoscopic pictures are viewed by the unaided eyes, they may be made to merge by the exercise of the requisite mental effort. In doing so the pictures will appear to move in their plane towards each other and, if the original separation is not too great, they will ultimately overlap and merge to form one picture in stereoscopic relief at an intermediate position. The observer feels his eyes, as it were, converging more and more towards this intermediate position. In reality his eyes are rotating outwards, not inwards, as will be evident from a consideration of Fig. 8, where $A$ and $B$ are the stereoscopic objects under observation. The eyes $C$ and $D$ naturally accommodate themselves to the distance of the object and in doing so they converge towards the intermediate point $C'$, the convergence being a function of the accommodation. The retinal images of $A$ and $B$ fall on dissimilar points and are accordingly separately appreciated as two images. By the exercise of mental

effort the eyes can now be rotated outwards without alteration of the accommodation until, when they are directed towards *A* and *B* respectively, the images fall upon the corresponding foveal points in which case *A* and *B* appear merged, being viewed simply as a single object lying apparently at the position *C'*, the natural fixation point for the particular condition of accommodation.

There are many who have not specially considered the matter who fail to realize that their eyes are not tending to converge when they make two separate objects merge in the manner described.

Before proceeding farther, it may be well to state that the two eyes of an observer are generally not equal in all respects. One of the eyes frequently controls the vision but, except where otherwise specified, it will be assumed that they share the work equally and that they rotate equally with respect to the median plane, which is not always the case.

It will be of interest now to reconsider Wheatstone's original experiments in the light of the relationship of convergence to accommodation. His illustration, Fig. 3*, is reproduced in Fig. 9, descriptive letters only having been added. According

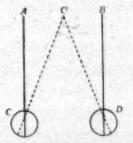
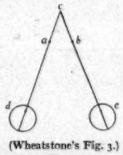


Fig. 8



(Wheatstone's Fig. 3.)
Fig. 9

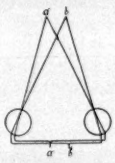
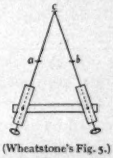


Fig. 10



(Wheatstone's Fig. 5.)
Fig. 11

to Wheatstone the axes of the eyes converge through the objects *a* and *b*, the images of which fall on corresponding retinal points giving rise to a single image situated at the intersection *c* of the axes. The diagram is one that might very readily be accepted by the uncritical reader but it is only necessary to repeat this and other of Wheatstone's experiments to realize the inadequacy of his descriptions.

Normally the unaided eye will see the two objects directly and no question of stereoscopy will arise, as is evident from Fig. 10, in which the images of *a* and *b* fall on corresponding points of both retinae and give rise to two mental images *a'* and *b'*. Suppose, however, that by the exercise of mental effort a position *c* is fixed. Then, as already explained in connection with the previous Fig. 8, the objects *a* and *b* will appear to move together and ultimately merge, in which case the axes will be directed as in Wheatstone's diagram. But the combined object will not be at the position *c*, as stated by Wheatstone. It will be in the plane of *a* and *b*. Its distance will not be altered by the act of convergence.

Wheatstone* states that "as both these modes of vision are forced and unnatural, eyes unaccustomed to such experiments require some artificial assistance." The simple arrangement of tubes used by him for this purpose is illustrated in Fig. 11. If this experiment is repeated, it will be found that, as the convergence

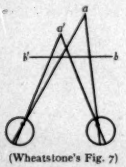
* *Scientific Papers*, p. 228.

is varied, the position of the resultant image does not alter by an amount at all comparable with that required to satisfy Wheatstone's diagram. As the tubes are brought more nearly parallel, the intersection of the axes and with it the image should recede. The apparent displacement of the image is actually a small amount of transient duration that may be attributed to a momentary change of the accommodation as the convergence is altered.

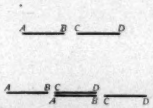
Wheatstone's seventh figure, reproduced in Fig. 12, is of particular importance, as it describes the synthetic formation of two stereoscopic images from four objects.

The stereo images *a* and *a'* are represented as both lying behind the plane *bb'* by an amount that may become infinite. But, if the experiment is repeated, it will be found that one of the stereo images always lies in the plane of the objects and more careful observation will reveal the existence of certain limits to the distance of the other image from the plane *bb'*.

For a truer understanding of the experiment it is necessary to proceed as before from the condition that the convergence of the eyes is normally determined by the condition of accommodation. If this is done there is obtained an entirely different



(Wheatstone's Fig. 7)
Fig. 12



Figs. 13a & 13b

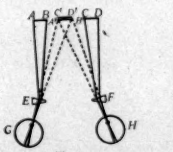


Fig. 14

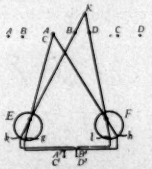


Fig. 15

diagram that makes it possible to prophesy what are the directions in which a stereoscopic image will move as the separations of the object marks are varied.

The phenomena of stereoscopic vision will, therefore, now be considered from the point of view—which I regard as fundamental—that the convergence of the unaided eyes is principally determined by their state of accommodation, and that mental effort or other aids are necessary to alter their relationship.

With practice a pair of ordinary stereoscopic photographs can be merged without the use of a stereoscope. The pictures appear to move towards each other and ultimately, if sufficient mental effort is exercised, they combine to give a single stereoscopic view. But the observer actually sees the combined pictures flanked on either side by the original and now separated pictures.

Thus, suppose the two views are represented by the lines *AB* and *CD*, Fig. 13 *a*. When viewed stereoscopically three pictures will be seen by the observer, namely a central combined stereoscopic picture flanked by the two original views, which now appear separated, as indicated in Fig. 13 *b*. The explanation is the simple one previously illustrated by Fig. 8, that, although the eyes have rotated outwardly in order to view the pictures axially, the combined image appears to lie at the central position corresponding with the point of convergence for the particular condition

of accommodation. By suitable screening, the simple flank images can, of course, be eliminated.

Mental effort may be avoided by the use of various aids such as the prisms *E* and *F* of Fig. 14, placed in front of the eyes *G* and *H* which, although naturally converged, view the objects *AB* and *CD*, the combined image of which appears to lie at the intermediate position.

In reality these intermediate images are not strictly parallel to one another, their inclinations being those attributable to the swinging, as it were, of the objects from the flanking positions to the intermediate position. But small variations of this kind will be neglected in the consideration of subsequent experiments, as the errors introduced thereby have been found to be quite inappreciable. It will also be assumed in the experiments to be described that the objects have been merged and brought into the intermediate position.

In Fig. 15, *AB* and *CD* are two pairs of separated objects indistinguishable from one another in form, and the points *AC* have been fixed by the eyes *E* and *F* and caused to merge. They are accordingly represented as one point. Images of *B* and *D* fall upon corresponding points and give rise to a single image situated apparently at *K*. If this Fig. 15 is compared with the earlier Fig. 12 of Wheatstone

Fig. 16

Fig. 17

Fig. 18

it will be realized how essentially different are the results obtained from the two diagrams. When *CD* is greater than *AB*, the point image *K* lies beyond *AC* and it is said to appear orthostereoscopically; as *D* is moved towards *C*, the point *K* advances along the definite direction *BK*, as will be very evident to an observer who repeats the experiment. When *D* reaches *B*, that is, when the separation of the pairs of points *AB* and *CD* are equal, *K* will coincide with *B* and two simple images will appear in the fixation plane containing *A*. This stage is represented in Fig. 16, in which only the principal rays corresponding with Fig. 15 have been indicated. As *D* moves still farther towards *C* the stereoscopic image *K* will now lie on the near side of the fixation plane, as in Fig. 17. It is said to appear pseudo-stereoscopically, but the transition is so regular and normal that the use of these terms will be avoided, as being unnecessary and indeed rather meaningless.

Suppose now the separation *CD* becomes zero; the image *K* should approach the position $K_1$, Fig. 18. But it is obvious that an object at *A* and one at *B* will appear simply as two objects in the plane *AB* in accordance with the earlier Fig. 16, and not as an image at *A* and a stereoscopic image at $K_1$. There must be a position of inflection or a failure of the stereoscopic phenomenon in the region indicated. That there is a failure is evident if we follow the process still farther until *D* has proceeded beyond *C*, as in Fig. 19. Since *A* and *C* fall upon the foveae, they form a simple image, but the images of *B* and *D* now fall on disparate points *g* and *h*.

They must be seen separately and cannot give rise to a single stereoscopic image at $K_2$. A further motion of $D$ towards the left could never again result in a stereoscopic image, as disparate points would continue to be excited. When the experiment is made it will be found that, as $D$ approaches $C$, say from the right, stereoscopic vision does actually break down at a definite distance of $D$ from $C$. The eye becomes confused and sees the images break up and change their positions, as if unstable. As the displacement of $D$ is continued, the confusion persists until, when $D$ has passed to the other side of $C$ by a certain amount, the stereoscopic image reappears, but its direction of motion will now have changed by an amount that is very recognizable in the field of view.

The explanation is really simple. During the period of confusion the fixation of the eyes wanders indefinitely between $A$ and $B$. When the point $D$ passes beyond $C$ to the critical point, the fixation of the eyes changes from $A$ to $B$, and the diagram now becomes that indicated in Fig. 20.

The fixation has shifted from $A$ to $B$, which now coincides with $C$, the images of which fall on the foveae and produce a single impression. The images of $A$ and $D$

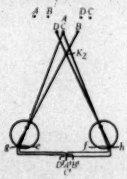


Fig. 19

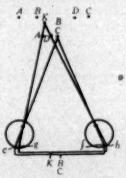


Fig. 20

Fig. 21

Fig. 22

fall on corresponding points and give rise to a single stereoscopic image at $K$ lying beyond the plane of $A$. Thus during the critical period of confusion the position of the stereoscopic image $K$ has shifted from the near side of the fixation plane to the far side.

Now as $D$ continues its movement farther towards the left away from $C$, the point $K$ approaches $A$ along the new direction $Ag$. When $CD$ again equals $AB$, the points $K$, $D$ and $A$ all coincide and two simple objects are seen in the one plane, as is to be expected.

Further movement of $D$ towards the left brings the single stereo image $K$ in front of $A$ as in Fig. 21. It will then be found by experiment that there is another limiting position of $D$ towards the left at which the stereoscopic image $K$ fails by breaking up into separate non-stereoscopic images of $A$ and $D$. Beyond certain limits the eyes cannot hold the images in combination. Similarly there is an extreme critical position of $D$ at the right hand side, as indicated in Fig. 22, representing the earlier condition in which $A$ and $C$ were fixed.

For pairs of objects all of which are similar in form arranged side by side, there are accordingly four critical points at which stereoscopic vision fails. They are as follows:

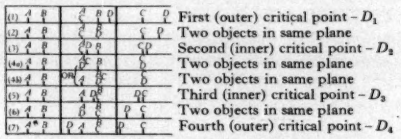
An extreme outer position on the right hand side, as in Fig. 22.

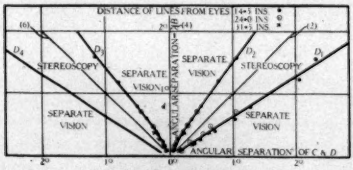
An inner position, as in Fig. 17, where the stereo image lies on the near side of the fixation plane.

A second inner position, as in Fig. 20, where the stereo image lies on the far side of the fixation plane, the fixation point being now altered.

A second extreme outer position on the left side, as in Fig. 21.

Before proceeding farther, it may be well to consider this question of critical regions from the quantitative point of view, but it will hardly be possible in the time available to do more than illustrate a few typical sets of observations. It must suffice to consider the results obtained from the combination of short and very fine pairs of lines. Thick lines gave similar results except at very small separations, the angular values of which were comparable with the angular widths of the lines.


(1) First (outer) critical point – $D_1$
(2) Two objects in same plane
(3) Second (inner) critical point – $D_2$
(4a) Two objects in same plane
(4b) Two objects in same plane
(5) Third (inner) critical point – $D_3$
(6) Two objects in same plane
(7) Fourth (outer) critical point – $D_4$




Limiting Stereoscopic Angles

Fig. 23

Fine dots and coarse dots gave curves of a type very similar to those obtained from fine and coarse lines respectively. Varying the length of the lines did not appreciably affect the results. Quantitatively certain of the results depend upon the particular observer.

A typical set of observations is represented by the curves of Fig. 23 which really comprise three sets of observations with the eyes accommodated for different distances. The dots represent readings on objects at 14·5 inches distance, the circles on objects at 24 inches, and the crosses on objects at 31·5 inches. As all the readings fall with considerable exactitude on fair curves, it will be evident that the positions of the critical points of stereoscopic vision are practically independent of the state of accommodation of the eyes and they are also independent of the convergence which is so closely associated with the accommodation.

In Fig. 23 the pairs of objects *AB* and *CD* are shown near the top of the diagram with the combined images between them. As the positions of each set of four

critical points depend upon the relative separations of $A$ from $B$ and $C$ from $D$, the sets of four critical points have been determined for various separations of $A$ from $B$ which during each set of observations were kept constant. It will be assumed that $D$ progressively approaches $C$ from the right. The first critical point occurs when $CD$ is considerably greater than $AB$, as indicated at (1), Fig. 23, and by the curve $D_1$. When $CD$ equals $AB$, the two images appear in the fixation plane of $AB$. The second critical point occurs when $D$ is close to $C$, as indicated at (3) and by the curve $D_2$. When $C$ and $D$ coincide, as at (4), the two objects appear in the fixation plane of $BC$. When $D$ has passed to the left of $C$ by a small amount, there occurs the third critical point, as indicated at (5) and by the curve $D_3$. When $CD$ equals $AB$ the two objects again appear in the fixation plane containing $BC$. When $D$ has passed considerably to the left of $C$ there occurs the fourth and last critical point $D_4$ as indicated at (7) and by the curve $D_4$.

In the diagram of Fig. 23 abscissae of the curves represent the angles subtended by the objects $C$ and $D$ and ordinates represent the angles subtended by the lines $AB$.

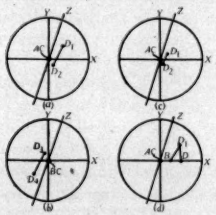


Fig. 24

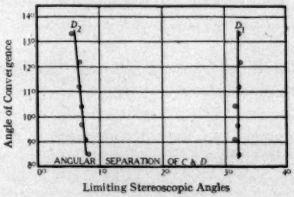



Fig. 25

Suppose, for example, the angular separation of $A$ and $B$ is 1° and the separation of $C$ and $D$ is varied as already described. When this separation is greater than 1·6° there is no stereoscopic vision. $C$ and $D$ are seen separately and, further, they are seen distinctly, which shows that the outer critical point $D_1$ is not determined by any question of indistinct vision. When the separation is 1·6° the images can be combined to appear as a single stereoscopic image.

Stereoscopic vision continues until the separation is reduced to 0·6°. Thereafter with astonishing abruptness stereoscopic vision fails and the vision becomes stereoscopically confused until $D$ has passed to the other side of $C$ and the angular separation has become − 0·6°, at which point the objects again combine to give a stereoscopic image. As the negative separation increases, stereoscopic vision persists, until it breaks down when the separation is − 1·6°.

If the angular separation of the lines $AB$ is a quarter degree, then the outer limits of stereoscopic vision are only ± 15 minutes, and the inner limits ± 6 minutes. A clearer idea of the meaning of these curves may possibly be obtained from the perspective diagram, Fig. 24, which represents the appearance of the field of view, the centre of which corresponds with the point of fixation and is, therefore, occupied

by the combined image of $A$ and $C$ or of $B$ and $C$, as the case may be. Vertical and horizontal diameters are indicated to emphasize the appearance of nearness or farness of points on the oblique line in the rectangular $XZ$ plane of co-ordinates. Thus in Fig. 24 *a*, $D_1$ and $D_2$ are the outer and inner critical points when $A$ and $C$ are fixed. Between the positions $D_1$ and $D_2$ there is stereoscopic vision and similarly in Fig. 24 *b*, in which $B$ and $C$ are fixed, there is stereoscopic vision between $D_3$ and $D_4$. Between $D_2$ and $D_3$ there is confusion as the point of fixation is being changed. Beyond $D_1$ and $D_4$ there is separate vision. The object $D_1$ beyond its outer critical position breaks up into the two original objects $B$ and $D$ and in doing so the two objects appear to move along the lines $D_1B$ and $D_1D$ of Fig. 24 *d*.

When the separation of $AB$ is made small, as in Fig. 24 *c*, it will be observed that the stereoscopic interval $D_1D_2$, and similarly $D_3D_4$, becomes correspondingly small.

It was stated in the earlier part of this paper that the phenomena of stereoscopic vision are practically independent of the condition of ocular convergence. In support of this conclusion the results indicated in Fig. 25 may be of interest, but, owing to the limitation of time, it will not be practicable to describe the means by which these and other results were obtained and a very brief reference to the results must suffice.

The convergence of the eyes was varied to suit the accommodation and the angular positions of one pair of the outer and inner critical points are those represented in the diagram. Practically the values are constant for all conditions of convergence and for all conditions of accommodation. Sir David Brewster, it will be recollected, regarded stereoscopic vision as being determined in some way by the convergence of the eyes which he assumed rapidly changed their fixation from the one object to the other and this view he upheld with all the vigour for which he was remarkable.

But Dove effectively disposed of this theory by demonstrating that objects were seen stereoscopically when illuminated by an electrical discharge the duration of which was a small fraction of that required for any definite change of the convergence.

That the outer critical point is not determined by the limit of distinct vision hardly needs demonstration since all those included in the diagrams of this paper have resulted from the excitation of points comprised within the macula lutea. Variation of the illumination has some effect upon the outer critical points $D_1$ and $D_4$ but no apparent effect upon the inner critical points $D_2$ and $D_3$. This will be evident from Fig. 26, which comprises the results of three sets of illumination experiments in which (1) both pairs of images were brightly and equally illuminated; (2) the illumination of both pairs was very dull although equal; and (3) one pair was brightly illuminated in comparison with the other.

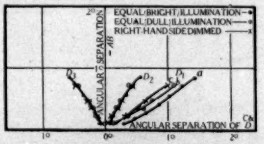
So far as the positions of the inner critical points $D_2$ and $D_3$ are concerned, variation of the illumination has no effect; the three curves closely agree with one another. But there is quite an appreciable effect in the position of the outer critical points. Strong illumination extends the limit, as indicated by curve (*a*), weak

illumination reduces it, as indicated by curve (*b*), and unequal illumination has a still greater effect. From curve (*c*) it would appear that the limiting points are determined by the low illumination of one of the pairs. The corresponding curves for the $D_4$ critical points are not indicated in the diagram. .

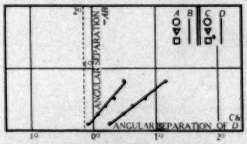
To affect the position of the inner critical point curves $D_2$ and $D_3$ does not appear to be easy. Independent variation of the powers of the two eyes has no appreciable effect. Curves obtained by young and elderly people were practically identical and variation of the colour of the objects produced no marked effect, but further research regarding these and other details is very desirable.

In all the preceding experiments the objects observed in each particular experiment have been equal in size and form. Those described were mostly made with short and very fine lines in order to ensure accuracy when the separations were small. Similar results, however, were obtained from observations on sets of long thin lines and on fine points. When observing on thick lines or spots the observations were affected at small separations, as was to be expected.

It is desirable now to consider certain important questions that arise when pairs of dissimilar objects are observed, a condition of affairs that corresponds more




Limiting Stereoscopic Angles
Fig. 26




Limiting Stereoscopic Angles
Fig. 27

nearly with natural conditions. In the first instance it will be assumed that the objects all lie on one straight line and that accordingly the images appear to move in one plane, as in the previous experiments. Inequality of size rather than of form appears to have the greatest effect in preventing stereoscopic combination of the images. Results obtained from the observation of pairs of objects comprising a fine line and a circle, triangle, and square are indicated in Fig. 27.

No difficulty need be experienced in combining the objects *A* with the similar objects *C* or the line *B* with the line *D*.

Provided the angular dimensions are comparable, there is also little difficulty experienced in combining the circle with, say, the triangle or the square, when they are arranged on the same horizon.

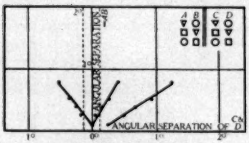
But the fine line *B* cannot be combined with the objects *C* and therefore after the inner critical point has been reached stereoscopic combination of the images is not again attained.

From a comparison of the curves of Fig. 27 with those previously shown, such for example as those of Fig. 23, it will be seen that the critical point curves now lie on one side only of the median line. The corresponding curves on the other side resulted in the previous experiments from the change of the fixation from *AC* to

*BC*. As the dissimilar objects *B* and *C* cannot be stereoscopically combined, the existence of only one pair of curves might have been anticipated.

Measurements of the angles subtended were made between the line and the centres of the objects *A* and from the diagram it would appear that the width of the object involves a displacement of the median line. While the occurrence of such a displacement is understandable, it is difficult to explain why it should be approximately equal to the angular width of the object *A* which had a linear width of 0·05 inch and a distance of 21·5 inches.

From observation on dissimilar objects of approximately equal size, such as those represented in Fig. 28, two sets of curves were obtained, as when observing on a pair of lines. Evidently the eye is capable of combining dissimilar objects under certain conditions, as for example those of equality of size. It was remarked, however, that, although the objects were combined stereoscopically, the resultant




Limiting Stereoscopic Angles

Fig. 28

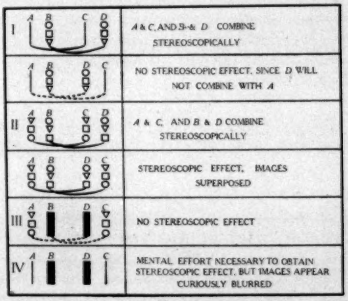

I
A & C AND B & D COMBINE STEREOSCOPICALLY
NO STEREOSCOPIC EFFECT, SINCE D WILL NOT COMBINE WITH A
II
A & C AND B & D COMBINE STEREOSCOPICALLY
STEREOSCOPIC EFFECT, IMAGES SUPERPOSED
III
NO STEREOSCOPIC EFFECT
IV
MENTAL EFFORT NECESSARY TO OBTAIN STEREOSCOPIC EFFECT, BUT IMAGES APPEAR CURIOUSLY BLURRED

Fig. 29

image appeared confused. To discriminate the details of its form was not easy. As in the previous illustration the median line appears to be displaced for each pair of curves, that is, towards the right and the left, and it is remarkable that the amount of the displacement is approximately 8 minutes, as before.

Some of the results obtained from observations of dissimilar objects are summarized in Fig. 29 and of these Nos. III and IV are of particular interest, as they indicate that a thick broad line will not combine with objects of equal angular width and that mental effort is required to combine a thick broad line with a thin line.

Natural conditions are still more nearly approximated to if the objects of each pair are on different horizons. Hitherto those considered have been on the same level and the combined images in approaching each other have intersected or interfered with one another.

In Fig. 30 the objects *A* and *C* lie on the same horizon, that is, on a line parallel to the ocular base, and the objects *B* and *D* are on another level. Under these conditions it is only possible to combine *A* and *C* or *B* and *D*, that is, objects on the

same horizon. For simplicity the case in which $B$ is brought directly above $A$ will alone be considered. When the general principles involved are recognized, it is quite easy to draw the diagrams for any other case of the same type and to predict the resultant stereoscopic appearances.

On the one retina $B$ lies directly above $A$ and $D$ above $C$ but to one side. The stereoscopic image of $BD$ in Fig. 30 lies above and beyond and towards the right of the combined image $AC$. If $D$ also lies directly above $C$ as in Fig. 31, then the stereoscopic image $BD$ lies directly above $AC$ and in its plane. If now $D$ lies on the left hand side of $C$ as in Fig. 32, the stereoscopic image $BD$ lies above, on the near side, and towards the left of the stereoscopic image $AC$.

Since in all three cases the object $BD$ lies above $AC$, with which it cannot interfere, it will be realized that there can be no inner critical points and the truth of this deduction can be confirmed by experiment. When, however, $D$ is moved sufficiently far towards the left or the right, stereoscopic vision fails and the two extreme critical points are obtained as in the other series of experiments. Thus for pairs of

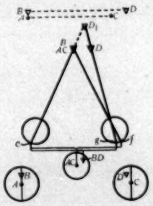


Fig. 30

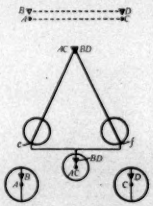


Fig. 31

objects on different levels there are only two critical points, namely the outer ones, $D_1$ and $D_4$.

But in a well-adjusted stereoscopic rangefinder it is usually the case that no critical points are observable when the stereoscopic images do not lie on the same level. This is due to the angular displacement necessary to reach either of the outer critical points being less than the total angular displacement of the images in the field of view that is attainable in the rangefinder. It is quite possible, however, so to adjust a stereoscopic rangefinder that one of the extreme critical points can be observed.

Reverting again to Figs. 30 and 32, it will be seen that a retinal image $B$ on the vertical meridian corresponds with a retinal image $D$ that lies in one case, Fig. 30, on the left of the vertical meridian through $C$ and in the other case, Fig. 32, on the right. This is a condition of affairs that is of importance in any discussion of the question of corresponding points. There is indeed no peculiar virtue associated with a vertical meridian passing through the fovea centralis and so far as stereoscopic vision is concerned it is improbable that there is any structural transition across the section in question. This conclusion was confirmed by a series of experi-

ments in which objects viewed obliquely were successfully combined stereoscopically. None of the images contributing to the stereoscopic appearance lay, in these experiments, on either fovea centralis or on verticals passing through them.

It is only for convenience of description that in the majority of cases direct vision experiments have been considered. Such oblique stereoscopic vision, however, is limited to comparatively small obliquities of about 1° 30′ to 2°. All the images, therefore, fall within the macula lutea.

Fig. 33 is of particular interest, as it shows the extreme positions of the outer and inner critical points of stereoscopic vision for similar objects on the same horizon. Only one pair of curves is indicated in the diagram, from which it will be seen that the two curves tend to meet. Thus at the limit the outer and inner critical points coincide and for further separations of $A$ and $B$ there is no stereoscopic vision possible.

That stereoscopic vision should be impossible over so large a proportion of the diagram is somewhat surprising but, if pairs of vertical lines that subtend angles

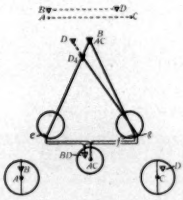
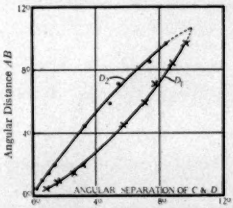


Fig. 32




Outer and Inner Critical Points
Fig. 33

within the critical areas indicated are viewed, the reality of these critical points will be soon appreciated, as a feeling of discomfort will be experienced in the attempt to combine the images.

It is possible that some observers may have more extended limits than those indicated, but the variations do not appear to be great.

I do not know if the opinion is still generally held that corresponding halves of the retinae are grouped together, as is suggested by the partial decussation of the optic nerve fibres that takes place at the optic chiasma, and that the nerves from corresponding elements of the two retinae are joined to one cerebral cell. Any explanation of the cerebral mechanism of vision appears to be rendered still more incomprehensible by the considerations that an element of one retina corresponds with any number of elements on the same horizon of the other retina; that retinal elements on either side of a vertical meridian through the optic axis of one eye may correspond with an element on the vertical meridian through the optic axis of the other eye; and that stereoscopic vision is possible under conditions of oblique vision, although limited to a few degrees. These considerations appear to make the

mechanism of stereoscopic vision even more inexplicable when considered on the basis of Hering's Cyclopean eye. But, although in the present state of our knowledge it is undesirable to speculate, the evidence seems to indicate as the seat of stereoscopic vision some cerebral position corresponding in effect with the fundamental conception of a Cyclopean eye.

Such a position is the optic chiasma regarding which but little is really known, notwithstanding that the details of the decussation of the optic fibres are generally indicated with a precision comparable with a plan of a railway junction. Conceivably the optic chiasma is something more than a mere junction for the distribution of the optic nerve fibres.

Such indefinite knowledge of the cerebral mechanism as at present exists is largely the result of clinical observation and pathological anatomy. Progress on these lines alone must necessarily be extremely slow and it is desirable, therefore, that for his guidance the pathologist should be supplied with the fullest details of the various phenomena of vision. These details can only be elucidated by continued research on the part of opticians and, if this contribution to the subject has served to demonstrate that the field of vision is still a great and unexplored one, the work of preparing this paper will, so far as I am concerned, have been justified.

## DISCUSSION

Prof. Cheshire: Although I agree with much that Dr French has said, there is still a great deal with which I do not agree, and which I think is either wrong or misleading.

As regards the earlier part of the paper there is little to say—the Horopter does not interest me much, because the angular field of acute vision, around the visual axes of the eye, is so extremely small that whether objects imaged appreciably off the axes are imaged singly or doubly, does not much matter if they cannot be seen at all.

Dr French has reproduced in his Fig. 9 the Fig. 3 of Wheatstone's paper. Now Wheatstone in his classical papers, which have been referred to, showed for the first time how the methods of descriptive geometry, invented by Monge, could be applied to the problems of stereoscopic reconstruction. This method, which has been followed without question or challenge by every later writer of repute, has received its fullest application, perhaps, in a booklet by M. Cazes called *Stéréoscopie de Précision*, and in a paper read before the Royal Photographic Society by Mr Lyndon Bolton in 1903 on the subject of "Stereoscopic Distortion." Now let us examine Wheatstone's figure. It shows points *a* and *b* with a certain separation in the plane of projection being observed binocularly, with the axis of the left eye directed through *a*, and the axis of the right eye similarly directed through *b*. Wheatstone states that under these circumstances the observer will see the stereoscopic image produced as though it were at the point *c*, the point of

intersection of the visual axes. In other words, according to Wheatstone, the two points *a* and *b*, which may be looked upon as the projections in the plane *ab* of an object point *c*, from the two eye points, will, if observed with the eyes converging as shown, fuse to present a single point seen as though at *c*. Dr French apparently agrees that the axial rays from the points *a* and *b* reach the left and right eyes respectively as though they proceeded from a single point *c*, but he says: "The combined object will not be at the position *c* as stated by Wheatstone. It will be in the plane of *a* and *b*. Its distance will not be altered by the act of convergence." This is a statement which it is very difficult to understand. Presumably, however, Dr French means that the axial rays from the single image *c* seen, enter the eye as from a single point in the plane *ab*. It is a pity that Dr French does not give a ray diagram to explain his contention. What is wanted is the ray diagram for the case in which, to use his own words, "The combined object" is not "at the position *c*" but "in the plane of *a* and *b*." In the absence of such a diagram further criticism would be futile, but it may be pointed out that, according to Wheatstone, the fusion of any two separated points in the plane of projection must give rise, under the conditions contemplated, to the impression of a single point from which the rays entering the eyes diverge under a smaller angle than they would do from a point in the plane of projection and thus the single image point must be seen as though behind any single point binocularly seen in the plane of projection*. Special stereograms to illustrate this point have been known for many years.

It is obvious that according to the principles of descriptive geometry an object-point represented in the plane of projection by a single common point must itself be in the plane of projection.

The inability of the eyes to estimate distance by the binocular observation and fusion of a single pair of points was well known to Wheatstone. The eyes demand the "referent plane" to which he makes constant allusion. A false stereoscopic character can be given a pair of similarly cut circular or square prints taken from the same photographic negative, but mounted with a smaller separation from centre to centre than that of any pair of corresponding points in the two pictures. This "fake" was exploited commercially on a large scale half a century ago and is still effective—incidentally also against Dr French's contention dealt with above.

Lt.-Comdr. H. Hickling: Dr French's contribution is a very valuable one, for at present we know very little about stereoscopy. Good stereoscopic vision depends upon a great number of factors, both physical and temperamental. For instance, expert observers found it impossible to take reliable readings with a stereoscopic rangefinder after they had run for several miles, though they were able to range accurately with a coincidence rangefinder. Observers suffering from toothache, headache or a cold had their stereoscopic vision impaired. There is little doubt that alcohol adversely affects the power to see stereoscopically.

* In the case of the stereoscopic fusion of the images of two points in the plane of projection with a wider separation than the interocular distance of the observer the geometrical method of reconstruction fails.

Mr J. Guild: There is one respect in which this paper differs from the previous papers by Dr French, that is, in the paucity of information concerning the experimental basis of the results. Without further details than are contained in the paper it is very difficult to follow what has been done.

Dr French contends that stereoscopy does not depend on convergence: but all that he has varied in his experiments is the actual convergence; he has not shown that the effect does not depend on the *difference* of convergence required for fusion of the respective pairs of points representing the nearer and further features of a stereoscopically perceived picture. It is on this difference—which is not, as far as I can see, affected in Dr French's experiment—and not on the actual convergence that stereoscopic relief depends according to any explanation of the phenomenon which I have seen advanced. Moreover Dove's experiment on instantaneous stereoscopic perception is not crucial, as it is not necessary where the estimation of some quantity—in this case *difference* of distance—is associated with a muscular movement that such muscular movement should be carried out; in some region of consciousness, or rather sub-consciousness, there resides the power, when the conditions defining two terminal states of a possible muscular movement are simultaneously perceived, of appraising the degree of innervation which would be required to produce the muscular movement between these states. It is this anticipatory appraisement of the required innervation which prevents overshoot and oscillation in all our muscular readjustments, and which, rather than the muscular movements themselves, conveys to us the information concerning the magnitude of the motion, either actual or potential.

I do not understand Dr French's reference, made while reading the paper, to parallax observations as a means of determining the position of a stereoscopic image in some of his experiments. Parallax observations will always give the true position of the object, or objects, in space, and will give no information concerning the *apparent* position of the stereoscopically fused image, which is purely an illusion due to the interpretation of observed effects in terms of the circumstances which would give rise to similar effects in the normal experience of the individual and the race.

One other point on which I would like Dr French's views concerns fixation. He frequently refers in the paper to the point of fixation and, I think, identifies this with the centre of the fovea. He appears to treat it as having a definite position and therefore as capable, in conjunction with other features of the eye, of defining directions and planes. Fixation is a very curious phenomenon: however close two points may be it is possible, provided they can be seen as separate points, to fix either of them but *not both*. In other words the area of fixation is of the order of that of a single cone. If, therefore, fixation is to be attributed to a retinal peculiarity we must assume that some one cone is differentiated in this very unique way from the thousands of others within the area of distant vision. This is not impossible but it seems highly improbable. My own view is that fixation is purely psychological: if two points are seen separately they give rise, of necessity, to separate percepts, and the mind can never have more than one percept in the

focus of attention at one moment. In other words we cannot give attention to more than one point of the multitude of separately distinguishable points making up a visual picture at any one moment, and it is this giving attention to a particular point which constitutes fixation. On this view the actual point fixed may lie anywhere on the retina provided it is within the relatively small region of distinct vision. I have had occasion to point out in connection with other visual peculiarities, e.g. the Fechner-Weber law of brightness perception, the great importance of distinguishing between properties requiring explanation on physiological lines, and those which are readily explicable on the elementary laws of psychology without invoking any assumptions concerning retinal structure. As I have never seen any attempt to account for fixation I would like very much to hear Dr French's views on the matter.

Comdr. Baker: While agreeing with a great deal of what Dr French states I think he is inclined to underestimate the importance of the convergence of the eyes as a factor in producing stereoscopic relief. He states "For the unaided eye accommodation and convergence are closely, although not rigidly associated. Normally a particular condition of accommodation occasions a particular convergence and to alter the convergence independently of the accommodation involves the exercise of mental effort or the use of artificial aids." My experience has been that there is very little interconnection between accommodation and convergence and this point is particularly exemplified in the case of an anaglyph. There accommodation is constant over the whole picture and is usually in my own case that associated with the near point. But the variation in convergence is very considerable as the eyes travel from one point of the picture to another while no special mental effort is necessary. The whole picture stands out boldly in stereoscopic relief from the first glance—*ça saute aux yeux*. If the close connection between accommodation and convergence exists as Dr French suggests, there would be an appreciable effort necessary to bring about the requisite convergence to change from a point in the foreground to one more distant.

Again if a spectacle prism is placed base outwards over one eye so as to bring about a considerable change in convergence, accommodation still works quite satisfactorily. I have tried this experiment with a 14Δ prism, equivalent to an 8° convergence on a distant object, and have had no difficulty in accommodating down to a distance of 70 cm. There is certainly a difficulty in seeing things closer than that distance but it must be remembered that with the prism the eye convergence then amounts to 14° or 15° and the difficulties are due to having reached the limit of convergence.

I am inclined to think that Dr French in elaborating his theories has not paid sufficient regard to convergence and particularly to differential convergence as the point of fixation changes from one object to another.

Prof. B. P. Haigh: I find nothing in Dr French's theory to contradict Wheatstone's theory, but very much to improve our knowledge of stereoscopy. The author has given us a very clear idea of the limits within which stereoscopic vision occurs.

Instr. Lt.-Comdr. N. M. S. Langlands: In experiments on depth perception with two rods which were placed at different distances and where observers were asked to state which rod was nearer it was found that in the case of 16 lieutenants qualifying in gunnery the average threshold angle (75 % of the estimates correct) was 3″ for binocular vision and 15″ for monocular vision. Using test plates in a stereoscope the average value was about 40″. In the practice of fusing two separated dots it was found that, when the separation approached the interocular distance, fusion usually became very difficult. If a glass plate were interposed and the image of a distant object brought into the same field as the dots, fusion became easy. This seems to show the close relation between convergence and binocular vision. That the stereoscopic rangefinder seems to fail when ranging at night on a light is perhaps due to the fact that the eye has no depth of field to converge over.

Dr French's results with objects not on the same level do not appear to agree with those of Messner and Volkmann.

Mr J. Rheinberg: Although I can readily fuse stereograms with the unaided eye, the fused images appear in such cases as slightly out-of-focus. In ordinary vision, too, blurred or out-of-focus images or objects are seen in stereoscopic relief (by short sighted persons for example) even though the two eyes may not be alike and the retinal images dissimilar in consequence. Experiments on those lines might help to solve the question referred to by Dr French of stereoscopic fusion of dissimilar shaped objects. When the retinal images approximated somewhat as with a triangle and a square or a circle of about the same size, the images might be capable of fusion although this could not be expected when they were too different, as in the case of a long line and a circle.

Capt. S. P. Holloway: Some years ago I was working with an oculist upon several devices for the restoration of stereoscopic vision to people who had lost it. We took a piece of green celluloid with holes punched in it to form a double cross and a piece of red celluloid with holes forming a single cross. These were superimposed and viewed through coloured filters—red before one eye and green before the other. The pieces of celluloid were then moved over each other until the single cross appeared inside the double one; this occurred at various positions of the actual devices, often with a separation of about 1¼ ins., and with noticeable obliquity.

The relation between convergency and accommodation can be varied but only to a limited extent; 4 or 5*D* is a usual limit of possible change of accommodation for a given convergence in the same person.

Von Graefe and Maddox found that fusion is destroyed by the dissimilarity of the objects: the latter distinguished between the functions of muscular balance and nerve reflexes. Have we here a case of true stereoscopic vision or simply coincidence of the optic axes helped by having objects of similar dimensions?

Dr L. C. Martin (communicated): The following impressions are left in my mind as a result of hearing Dr French's paper and I should be grateful for his comments on them.

(1) He speaks of Wheatstone's *discovery* that "the excitation of similarly situated but disparate points by an object not lying on the horopter may give rise to a simple sensation." Does he suggest that prior to Wheatstone's work it was seriously contended that in an object seen in stereoscopic relief one point only (or three points on the horopter) appeared single and all other points appeared double to the consciousness? This is a matter so easily tested by observation that I should be extremely surprised if the position of the "upholders of the horopter" has been correctly put.

(2) He has been at pains to emphasize the conclusion that the absolute convergence of the eyes has little or no influence on the stereoscopic sensation, but he has brought forth new evidence to support the view that it is (so to speak) the "differential coefficient" of the convergence which is the function concerned. Hence it appears to me to be highly dangerous to say that "the convergence of the eyes is a factor of but secondary importance." It is clear at all events that for the perception of stereoscopic relief between two points to be possible a necessary physical condition may be clearly defined, i.e. that, should the eyes pass from fixing one to fix the other, there must be a definite and often known minimum of change in angular convergence. The whole question is whether we are to regard this *condition* as the cause, and assert with Brewster that no stereoscopic relief is appreciated till the eyes have had an opportunity of assessing the alterations of convergence required. I shall be glad if Dr French will insert the reference to the work of Dove which, it appears to me, should be dealt with very much more fully if the above conclusion regarding convergence is to be substantiated.

From ordinary experience and observation it appears to me that much can be said for Brewster's contention. As in colour vision, the problem of attributing various psychological optical phenomena to the particular portion of the retino-cerebral mechanism concerned is one of great difficulty. At present (after hearing the paper) I should incline to the view that convergence, or its difference, is intimately involved in the mechanism of the main effects of stereoscopic vision, but in order to account for the results of Dove (if these are correct) it is necessary also to invoke a kind of "mental convergence" in which a variation of mental effort is required to associate pairs of images. There is no reason to conclude that the appreciation of stereoscopic relief must be instantaneous, for there would conceivably be time for the mind to test this mental convergence during the comparatively long period of the visual impression in Dove's experiments. Is there any reason why the two effects should not exist side by side? The ocular convergence may be likened to "sight," and the mental convergence to "faith." A little of the one marvellously helps the other.

In conclusion I regret that Dr French has not seen fit to provide a fuller description of what must have been very interesting experiments.

Dr French (partly communicated): I hope Prof. Cheshire does not advocate a laissez-faire attitude in his statement that a certain method "has been followed without question or challenge by every later writer of repute." It is often such taken-for-granted methods that require reconsideration. Although he prefaces his

remarks with the statement that there is "a great deal with which he does not agree and which he thinks is either wrong or entirely misleading," I can find in his remarks when carefully read no specific statement of his objections.

Wheatstone's experiments are, I think, inadequately described and, according to Prof. Cheshire, I have apparently committed the same fault with regard to Fig. 3 (my Fig. 9). My general statement, which has been quoted by Prof. Cheshire in support of that contention is followed, however, in the paper by the particular statement, which is as follows:

> The simple arrangement of tubes used by him (Wheatstone) for this purpose is illustrated in Fig. 11. If this experiment is repeated, it will be found that, as the convergence is varied, the position of the resultant image does not alter by an amount at all comparable with that required to satisfy Wheatstone's diagram. As the tubes are brought more nearly parallel the intersection of the axes and with it the image should recede. The apparent displacement of the image is actually a small amount of transient duration that may be attributed to a momentary change of the accommodation as the convergence is altered.

In these words I have endeavoured to express (I hope with sufficient conciseness and lucidity) the results, not of a theoretical discussion, but of a definite experimental observation.

If in Wheatstone's experiment the separation of the objects *a* and *b* is increased, does the resultant image *c* move progressively towards infinity? According to my observation, it does not.

Prof. Cheshire in his footnote removes the difficulty of what occurs beyond infinity by stating that "the geometrical method of reconstruction fails" in that region. A geometrical method that fails under such elementary circumstances is hardly worthy of the name. If the method described by me is followed, no difficulty of reconstruction is experienced, even if the eyes are assumed to be capable of divergence.

The last portion of Prof. Cheshire's remarks is devoted to a stereoscopic appearance of false character. However interesting optical illusions and "fake" stereoscopic appearances may be, they are not comprised within the subject of my paper and I cannot agree that deductions based upon fake monocular photographs can be advanced as an argument against the views expressed regarding binocular phenomena.

To Comdr. Hickling's very interesting practical observations I can add nothing except the statement that the adverse effect of nervous shock upon stereoscopic rangefinder observers was, I believe, experienced in the German Navy during the recent war.

Mr Guild has drawn attention to "the paucity of information concerning the experimental basis of the results." The apparatus employed was not of a very complicated nature. In the first instance simple figures drawn upon paper were combined by the unaided eyes. In the case of figures drawn upon squared paper, some curious results were obtained. They are not referred to in the paper, partly for want of space and also because I am still unable to interpret them.

I then used vertical wires mounted upon frames that could be displaced with reference to scales and in some cases micrometer gear. For general experiments

these wires sufficed. It was soon realized, however, that for measurements in the region of the inner critical points the angles subtended by the wires formed too large a proportion of the critical angles. Fine lines engraved and photographed on thin glass plates mounted in slides as described above were accordingly used for the more accurate measurements.

Mr Guild draws a distinction between actual convergence and "difference" of convergence, the meaning of which I possibly do not understand, since he considers I have not varied the "difference" of convergence, whereas I think I have varied all the elements. The term "convergence" I regard as referring to the inclination of the visual axes which intersect therefore at one point of the object referred to in my paper as the point of fixation. Rays from some other point of the object proceeding to the two eyes may have a different inclination, but, whatever that inclination may be, the convergence of the eyes is not necessarily affected. The angular separation of points of the object seen at one glance and falling within the area of the macula lutea have been altered in my experiments, the appearance of stereoscopic solidity being determined by such angular separations. I fear, therefore, I do not yet understand Mr Guild's argument. If the term "convergence" is limited in the manner indicated, Dove's experiment was, I think, crucial as a test of Brewster's opinion that in the act of appreciating stereoscopic solidity the eyes necessarily changed their convergence by fixing in rapid succession different points of the object.

Mr Guild has raised the interesting point as to whether or not parallax observations can be made between an object and a stereoscopic image. That is a question that can most easily be settled by actual experiments. Those made by me seemed to indicate that such observations are possible. Although I am not qualified to discuss the subject philosophically, might I venture to question the premises of Mr Guild's argument, which I understand to be as follows:

(*a*) Parallax observation is possible with reference to objects in space, presumably because they are concrete objects.

(*b*) Such observation is not possible with reference to a stereoscopic image because it is an illusion.

But the appreciation of an object in space results from the formation of an image on the retina and the transmission of the sensation to the brain where in some at present unknown manner there is formed a mental image. Surely such a mental image of a concrete object is as much an illusion as a stereoscopic image and therefore capable of being compared parallactically with the latter.

Again, suppose a stereoscopic image or illusion has been produced in the brain. The illusion is that of a particular solid object occupying a particular position in space. If, then, a concrete object is observed having the same form and occupying the same apparent position, the mental image of the object in space and the illusion should be indistinguishable and should presumably be comparable when they do not correspond exactly in form or position.

Mr Guild asks what are my views regarding the term "fixation" which I have used throughout the lecture. For purposes of description it has been necessary to assume at any one instant a definite convergence of the eyes upon some particular

point of the object under observation—the point of fixation. That on the retina "the area of fixation is of the order of that of a single cone" is, I think, very unlikely. A diameter comprising twenty cones is more probable. But, although very distinct vision may be possible over such an area, it is conceivable that in fixing an object the axes of the eyes may be determined within limits of much smaller dimensions.

To discuss fixation on psychological lines is, I fear, quite beyond me. Psychology is, I understand, the science of the mind or soul. Pliny tells us that "The Eies are the verie seat and habitation of the mind and affection." In those early times, therefore, the eyes were a fit subject for psychological consideration. To-day we know a great deal about the mechanism of the eye but very little about that of the brain, and, while things mental are still considered psychologically, things ocular are now considered physiologically. Some day we may know much of the details of the mental mechanism and presumably visual phenomena will then be considered altogether on a physiological basis, in which case the distinction emphasized by Mr Guild will disappear.

In reply to Comdr. Baker, may I repeat—possibly in other words—what I have already stated in the paper? Suppose two objects are arranged side by side symmetrically with the medial line and are combined visually with the resultant formation of a stereoscopic image in relief on the medial position. If then these two objects are separated while their distance from the eyes remains unaltered, the visual axes will alter their convergence. But, notwithstanding the change of convergence, the stereoscopic image will retain almost exactly not only its original form but also its original position on the medial line where the eyes would normally converge under the particular conditions of accommodation.

In the selection of an anaglyph for the illustration of his remarks, Comdr. Baker has chosen an object that has very little relief and for the moment I shall consider the object as being a simple plane vertical surface, set parallel to the ocular base. If the eyes wander over such a surface, the convergence will of course alter, but surely, since the ocular base is fixed and the points of convergence lie upon the surface, the distances of the points must alter with the convergence. Comdr. Baker's claim that, in viewing an anaglyph "the accommodation is constant over the whole picture" surprises me. As Comdr. Baker always focussed with care on these points, I presume his power of accommodation is not seriously restricted. I suggest, therefore, that the reason he experienced no mental strain was the natural one that automatically the accommodation and convergence altered to suit the position of the point of fixation at any particular moment.

I need only thank Prof. Haigh for his remarks, which call for no reply. He has stated what is also my belief, that the results of my experiments do not contradict in any way Wheatstone's great discovery of Stereoscopic Vision.

The title of my paper was intended to indicate that some re-statement of the grand unassailable principle was alone necessary, and I hope that feeling is justified by the experimental evidence produced and particularly that regarding the existence of critical points.

Comdr. Langland's data are very interesting and indeed remarkable so far as

the three-second threshold value for binocular vision under the conditions indicated is concerned.

The remarks of Comdr. Langland, Mr Rheinberg, and Capt. Holloway do not appear to call for any reply. I need only thank them for their interesting contributions to the discussion.

Dr Martin's first query refers to a question of history, the interpretation of which so often varies with the individual. The views regarding Wheatstone's "discovery" expressed in the paper and referred to by Dr Martin are the result of a careful consideration of the historical literature of the subject which I have been accumulating for a long time.

In view of the ease with which the matter "can be so easily tested," Dr Martin expresses doubt as to whether or not the position of the "upholders of the horopter" has been correctly put by me. While I do not say Sir David Brewster held any very strenuous views regarding the horopter, it is interesting to consider his action in the matter. He recognized that, contrary to the prevailing opinion, points of the stereoscopic image at apparently different distances appeared distinct at the same time, but he at once sought to bring the new evidence into line with the older conception by the explanation that the eyes fixed points at different distances in rapid succession, recognizing at any moment the singleness of the points fixed and neglecting the duplication of the others.

I have referred particularly in the paper to Prévost. What was his attitude? Although his famous paper was written after the publication of Wheatstone's discovery, to which he refers, he presupposes the idea that points not on the horopter, if appreciated, must appear doubled. New ideas have usually to fight for recognition and in this respect Wheatstone's discovery received no exceptional treatment.

With regard to Dr Martin's second point, I have already explained in my reply to Mr Guild what I mean by convergence and that I do not regard the angles of oblique rays (akin to angular field) as determining convergence. I do not think it is permissible to include questions of angular field under the term "convergence of the eyes," which should be restricted to the convergence of the visual axes.

Dr Martin will find the description of Dove's experiments in the *Berichten der Berliner Akademie* of 1841. There is also a brief account in Dove's own book, *Darstellung der Farbenlehre*. My copy, dated 1853, also contains Dove's *Optische Studien*, a great part of which is devoted to stereoscopy and particularly those opinions of Sir David Brewster to which he took exception.

Dr Martin seems reluctant to abandon convergence as a principal element of stereoscopy and this reluctance forces him to question the correctness of Dove's experiments, although actually unacquainted with them. If Dove's experiments should happen to be correct, Dr Martin would then invoke the aid of "mental convergence" and thus translate the problem from the prosaic observational plane in which my paper lies to the higher and possibly more fascinating psychological one, in which unfortunately I possess no degree of freedom.

It only remains for me to thank the members who have so kindly discussed the subject of the lecture and who in doing so have contributed so much additional and valuable information.

# REVIEWS

*Advanced Practical Physics for Students.* By B. L. WORSNOP, B.Sc. and H. T. FLINT, M.Sc., Ph.D. Pp. viii + 640; 394 Figs. (London: Methuen and Co., Ltd., 1923.) Price 21*s.* net.

In reviewing this treatise it is perhaps necessary to make allowance for the fact that the authors have attempted an impossible task. The volume and variety of knowledge—both of theory and technique—included within the scope of the word "Physics" has grown to such an extent in recent years that it is no longer possible for any one author—or pair of authors—to possess first hand knowledge and experience of more than a small section of the subject. When this becomes recognized we shall no longer have pretentious "Textbooks of Physics"—either theoretical or practical—which can never be more than mere scissors and paste collections of methods, most of them obsolete, beloved of the older teachers of physics and cheerfully perpetuated by many of the younger generation who apparently never go outside the walls of their Alma Mater in search of a little fresh air.

The present reviewer will not make the same mistake as the authors of the book by claiming the ability to review the whole of it, but will confine his attention to the section dealing with optics. In the first place the majority of the space is taken up with most elementary methods and explanations with which the student, according to the preface, should already be familiar. Thus "pin" methods of measuring focal lengths, curvatures etc., and experiments of similar accuracy, while admirably suited to an introductory course for schoolboys, should have no place whatever in an advanced practical textbook. Had this elementary material been omitted, space might have been found for one or two of the methods suitable for really advanced work and for a discussion of the precautions which have to be taken and the principles which have to be understood before precise measurements can be made. The student or other worker in search of information of this kind will not find it here. Thus in connexion with such a fundamental measurement as that of refractive index the only precaution mentioned is to avoid errors due to backlash in the tangent screw—which is the more unfortunate inasmuch as this is almost the only precaution which need not be taken in such measurements, backlash being obviously without effect on the readings, except in the case of particular instruments in which the tangent screw also serves as a micrometer. Then in describing the Newton's Rings method of curvature measurement, after mentioning the correct method of arranging the lens and the flat the authors discount this by asserting that the wrong method is more convenient and that the error involved is "not great."

In the section dealing with photometry the level of the work is that in which candles are used. The universally employed Lummer-Brodhun photometer is wrongly described, while in referring to flicker photometry an early and practically obsolete form is described and no reference is made at all to the circumstances—namely measurements involving colour difference—for which such an instrument is used.

Many of the explanations given are involved, and give an unnecessary impression of difficulty and complexity. The diagrams are in many cases so sketchy as to leave doubt on essential points, and in at least two cases they are quite misleading. In one of these (Fig. 169) the autocollimating method of measuring refractive index is illustrated as applied to what is clearly intended to be an ordinary equilateral prism. No indication is given in the text that the greatest refractive index which the prism could have in such circumstances would be in the neighbourhood of 1·15! In the other case (Fig. 235) the two wedges of a quartz compensating saccharimeter are shown with their bases opposite instead of with the base of one opposite the apex of the other.

From the optical section of this work at any rate one fails to find any justification for adding still another to the long list of books which clamour for the patronage of that overburdened and unprotected creature—the student.

J. G.

*Mirrors, Prisms, and Lenses, A textbook of Geometrical Optics.* By JAMES P. C. SOUTHALL. Pp. xix + 657; 287 Figs. Enlarged and revised edition. (New York: The Macmillan Company, 1923.) Price 15*s.* net.

The issue of a second edition of the above work (which was first published in December, 1918) appears to indicate that Prof. Southall has found this method of approach to geometrical optics sufficiently valuable to retain it without serious modification. The first edition dealt in some detail with rectilinear propagation, reflection, refraction, and applications of the fundamental principles to prisms, spherical refracting surfaces, and thin lenses; further chapters dealt with "curvature," astigmatic lenses, geometrical theory of the symmetrical optical instrument, compound optical systems, aperture and field, the eye, dispersion and achromatism, and aberrations. The present edition contains, beside a variety of new problems and exercises, an additional chapter giving an historical summary and a few supplementary paragraphs dealing, amongst other things, with some of the topics discussed during recent years in the pages of these *Transactions*. The work is most thoroughly done and the book should be extremely useful to optometrists as well as to general students.

It might be suggested, however, that the thoroughness of the book may be a little dangerous to the elementary students for whom it is written, those, at least, who desire to pass on to a study of optical designing and who cannot therefore stop short at the point where Prof. Southall quotes (perforce without discussion or explanation) the famous equations of Seidel. There is a danger of over-elaboration in the elementary stages of most subjects and it is not usually considered necessary to discuss "harmonic ranges" before proceeding to a first consideration of optical images. In an attempt to be clear the author has provided perhaps a little too much food for thought.

In the experience of the present writer, "ray tracing," as applied to lenses, ought to be introduced at an early stage. No apology is ever made in textbooks for the usual elaborate ray tracing applied to prisms and mirrors, yet when the student reaches the subject of lenses he is at once made to abandon exactitude in favour of approximation. This is no doubt inevitable in the very first introduction to the subject, but serious students always feel that they have no means of judging how far the approximations are applicable. In *Mirrors, Prisms, and Lenses* ray tracing for the case of spherical refracting surfaces is not dealt with until the end of the book.

It is difficult properly to appreciate the Gaussian theory unless it can be viewed against a background of the experience gained by the application of the exact law of refraction to a few simple cases. A telescope objective through which a few rays have been traced is a thing mastered and understood in a way which is otherwise impossible. From a short series of studies of this kind the student returns with enthusiasm and understanding to the more elegant algebraical and geometrical treatments.

L. C. M.

# ABSTRACTS OF PATENT SPECIFICATIONS

188434. Cambridge & Paul Instrument Co., Ltd., and others. FINE ADJUSTMENTS.

A disc controlling the setting of the instrument is operated by the rotation of a flexed wire which bears upon the edge of the disc. The device is described in connection with a moving-coil galvanometer, but is stated to be applicable to other physical instruments.

188721. Parsons, Sir C. A., and others. OPTICAL GLASS.

The melt is allowed to cool until it has set, while still remaining viscous, and the bulk of the material is then separated from the pot. One method employed consists in driving a steel tube with a cutting edge right through the mass of glass and the bottom of the pot, and then cutting away the disc of the pot material adhering. Or, the pot may be inverted, and heated externally to melt the outer layer of glass, and permit the residue to fall out. Thirdly, the material of the pot may be broken up, and the pieces extracted.

189081*. Nisot, F. STEREOSCOPIC APPARATUS.

A relief effect is obtained by projecting images of pictures which have been taken from more than one view point. An optical system is employed consisting of two fixed prisms, and a reciprocating frame bearing two other prisms, which operate each in conjunction with one of the fixed prisms, and come alternately into position before the objective.

189161. Bell, L., and another. SECRET SIGNALLING.

Morse signalling is effected by a projected beam of ultra-violet radiation, lying between the limits of visibility and of vitreous absorption. The same instrument serves as a transmitter, when it projects a beam of light filtered through superimposed sheets of purple nickel glass, and green copper glass; or as a receiver, when the light falling upon it is focussed upon a fluorescent screen, and observed by an eyepiece.

190997*. British Thomson Houston Co., Ltd. MEASURING RADIATION.

The actinic value of the radiation from a small source is determined by an instrument comprising a sighting tube, which is fitted with diaphragms to cut off extraneous energy, and a sensitive cell arranged in an electrical circuit.

191015*. Zeiss, C. SIGHT-TESTING APPARATUS.

Two partial images of a sighting mark are projected upon the retina of the subject by means of a system of prisms, and a Scheiner disc, and a telescope permits of the examination by an observer of the images so formed.

191117. Howland, A. COMPARING COLOURS.

For determining the hue, strength, and luminosity of colours, one or more charts are used in conjunction with discs comprising variously coloured sectors. Various systems by which the characteristics so determined may be indicated upon diagrams are described.

191331. Soc. d'Optique, &c. de Haute Précision. RANGEFINDERS.

Two parallel plates, which are used in achieving coincidence, are placed in one of the telescope systems, and rotated by bevel gearing, so that the two images of the object are always formed on the horizontal line passing through the centre of the eyepieces.

* Specification published before acceptance.

191754. Low, A. M. GUN SIGHTS.

Two telescopes are mounted side by side on the gun, one being anastigmatic for sighting purposes, and the other being astigmatic, for observing, with distortion, the bullet holes.

193036. Zeiss, C. STEREOSCOPIC APPARATUS.

In comparing by binocular observation the apparent luminosity of two marks movable in parallel planes, these marks are carried by sliders connected by a link in such a way as to move in opposite directions with respect to a definite boundary plane. The equality of the apparent change of depth of the two marks as they move away from the boundary plane indicates that the luminosity is alike to both eyes.

193078. Franz, L. AIMING-DEVICES.

A night aiming-device for firearms comprises an optical system which projects an image of the spiral filament of an electric lamp upon the target towards which a diffused beam is also directed by a reflector. Optical elements such as prisms may be fitted to reduce the over-all length, and the system may include an erecting device, so that the lamp may be replaced by an astronomical eyepiece for testing the system in daylight.

193969. Wright, E. H. LENS SYSTEMS.

An inner lens system is surrounded by an outer annular system which is capable of independent adjustment, so that objects in two different planes may be simultaneously focussed.

194996. White, A. E. MICROSCOPES.

The appliance consists of a ring-shaped device for the illumination of opaque objects. Light coming from beneath is reflected by an oblique face formed upon the outer edge of an upstanding flange, and refracted through the inner surface upon the object.

195660. Graves, A. K. PROJECTION APPARATUS.

The reflected beam of light in an episcopic system is made to conform closely to the rectangular shape of the object picture by employing a reflector with a central parabolic portion merging into a curve of a different eccentricity. A small bowl-shaped reflector intercepts the direct rays. The projector lens is stated to be uncorrected for actinic rays, so that flatter curves can be used.

195965*. Zeiss, C. OPTICAL SYSTEMS.

In a copying or projection apparatus, an auxiliary lens is mounted in front of the main lens, and is inter-connected with the receiving surface to move in such a way as to maintain the focus for different degrees of magnification.

R. J. T.

* Specification published before acceptance.